Heavy quarkonia in a medium as a dissipative quantum system: Master equation approach

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Heavy quarkonia in a medium as a dissipative quantum system: Master equation approach

- Heavy quarkonia in a medium as a dissipative quantum system?
- Master-equation approach to dissipative quantum systems
  - internal degrees of freedom
  - external degrees of freedom
- Master-equation approach to the real-time evolution of quarkonia in a QGP.

N.B. & C.Gombeaud, arXiv:1003.2945 + arXiv:1009.4271

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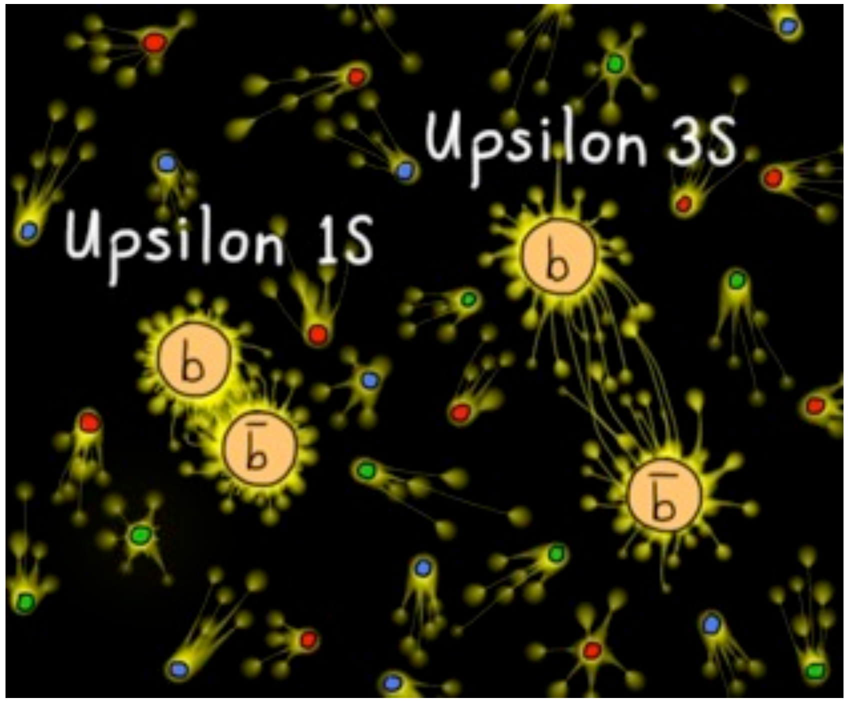
A naïve picture...

Quarkonia ➡ few internal degrees of freedom: "small system"

almost no influence



big effect

Quark-gluon plasma ➡ many degrees of freedom

Medium can transfer energy & momentum to the quarkonium without being significantly affected: **small system** in contact with a **reservoir**.

Paradigm setup of **dissipative quantum systems**.

☛ Might be useful to study the real-time dynamics of quarkonia.

Dissipative quantum systems: generic setup & properties

- Small system $\mathcal{S}$ + reservoir $\mathcal{R}$ constitute a closed total system:

Hermitian Hamiltonian $H = H_{\mathcal{S}} + H_{\mathcal{R}} + V$ ⇒ unitary evolution

($H_{\mathcal{S}}$: free small system; $H_{\mathcal{R}}$: free reservoir; V: interaction)

- The reservoir/bath dynamics are "uninteresting": the corresponding degrees of freedom are integrated out.

⇒ non-unitary effective evolution ($(H_{\mathcal{S}})_{\rm eff}$) of the small system:

open, dissipative quantum system.

Reservoir influence encoded in non-Hermitian $(H_{\mathcal{S}})_{\rm eff}$.

☛ see also next talk by Nirupam Dutta!

Dissipative quantum systems: time scales

$$H = H_{\mathcal{S}} + H_{\mathcal{R}} + V$$

free small system ($H_{\mathcal{S}}$) — free reservoir ($H_{\mathcal{R}}$) — interaction (V)

$V = S\,R$, where S acts on the small system, R acts on the reservoir.

In a large reservoir in a stationary state, the autocorrelation function $\langle R(t)R(t-\tau)\rangle$ takes non-negligible values only in a small interval around $\tau = 0$, of typical size τ_c.

☞ characteristic of the reservoir fluctuations

(Should not be resolved by the small system?)

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Master equation formalism

("Redfield formalism": Wangsness & Bloch 1953, Redfield 1957)

- Total system described by its density matrix $\rho(t)$

unitary evolution: $$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[H, \rho(t)]$$

- Integrating out the reservoir degrees of freedom:

$\Rightarrow$ reduced density operator $\rho^{\mathcal{S}}(t) \equiv \mathrm{Tr}_{\mathcal{R}}\big(\rho(t)\big)$

obeys a non-local evolution equation

- Neglect correlations between reservoir and small system & consider evolution of small system on time scales $\gg \tau_{\mathrm{c}}$.

$\Rightarrow$ local equation $$\frac{\mathrm{d}\rho^{\mathcal{S}}(t)}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[H_{\mathcal{S}}, \rho^{\mathcal{S}}(t)] + \mathcal{L}[\rho^{\mathcal{S}}(t)]$$

encodes properties of the bath and its coupling to the small system

Master equation formalism: ingredients

- The small system is characterized by the reduced density matrix.

  $\Rightarrow$ energy eigenstates

  Hereafter: diagonal elements $\rho^{\mathcal{S}}_{ii}$ ("populations") only.

- From the reservoir, only the average number of excitations $\langle n_\lambda \rangle$ in each mode λ is needed.

  $\Rightarrow$ can be a thermal bath, but not necessarily

- Interaction: rates $\Gamma_{i\to k}$ of the reservoir-induced transitions between states of the small system.

Master equation formalism: internal degrees of freedom

☞ neglect the motion of the small system

For the populations of the energy eigenstates, the master equation for the reduced density matrix yields coupled Einstein equations

$$\frac{\mathrm{d}\rho^{\mathcal{S}}_{ii}}{\mathrm{d}t}(t) = -\sum_{k\neq i}\Gamma_{i\to k}\,\rho^{\mathcal{S}}_{ii}(t) + \sum_{k\neq i}\Gamma_{k\to i}\,\rho^{\mathcal{S}}_{kk}(t)$$

IF the reservoir is a thermal bath at temperature T, then the rates satisfy $\Gamma_{i\to k}\,\mathrm{e}^{-E_i/k_BT} = \Gamma_{k\to i}\,\mathrm{e}^{-E_k/k_BT}$, and the populations tend to stationary solutions $\big(\rho^{\mathcal{S}}_{ii}\big)_{\mathrm{eq.}} \propto \mathrm{e}^{-E_i/k_BT}$.

☛ "equilibration of the internal degrees of freedom"

Master equation formalism: external degrees of freedom

☞ motion of the small system

The energy eigenstates are now labeled with their internal quantum numbers and their momentum: $\rho^{\mathcal{S}}_{ii,\mathbf{pp}}$.

Define then $\pi(\mathbf{p},t) \equiv \sum_i \rho^{\mathcal{S}}_{ii,\mathbf{pp}}(t)$, summed over all internal states.

- The rate of evolution of $\pi(\mathbf{p},t)$ is lower than those of the $\rho^{\mathcal{S}}_{ii,\mathbf{pp}}(t)$.
- When the internal degrees of freedom have equilibrated, π obeys a Fokker–Planck equation (for small momenta)

$$\frac{\partial\pi(\mathbf{p},t)}{\partial t} = \eta_D \nabla_{\mathbf{p}} \cdot \left[\mathbf{p}\pi(\mathbf{p},t)\right] + \kappa \triangle_{\mathbf{p}}\pi(\mathbf{p},t)$$

IF the reservoir is a thermal bath at temperature T, $\kappa = \eta_D M_{\mathcal{S}} k_B T$.

Master equation formalism: time scales

- Characteristic scale τ_c of the reservoir fluctuations.

$$\ll$$

- Typical scale for the evolution of the populations of internal states

$$\approx 1 \,/\, \Gamma_{i\to k}$$

$$\ll$$

- Characteristic scale for the evolution of the small system momentum

$$\approx 1 \,/\, \eta_D$$

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Master-equation-based description of quarkonia in a QGP

- Plasma modeled as a 3-dimensional reservoir
  1. "Gaussian bath" ($\langle n_\lambda \rangle$ peaked around some energy)
  2. Thermal bath

- ... which induces vector transitions (through dipolar coupling) in the $Q\bar{Q}$ system

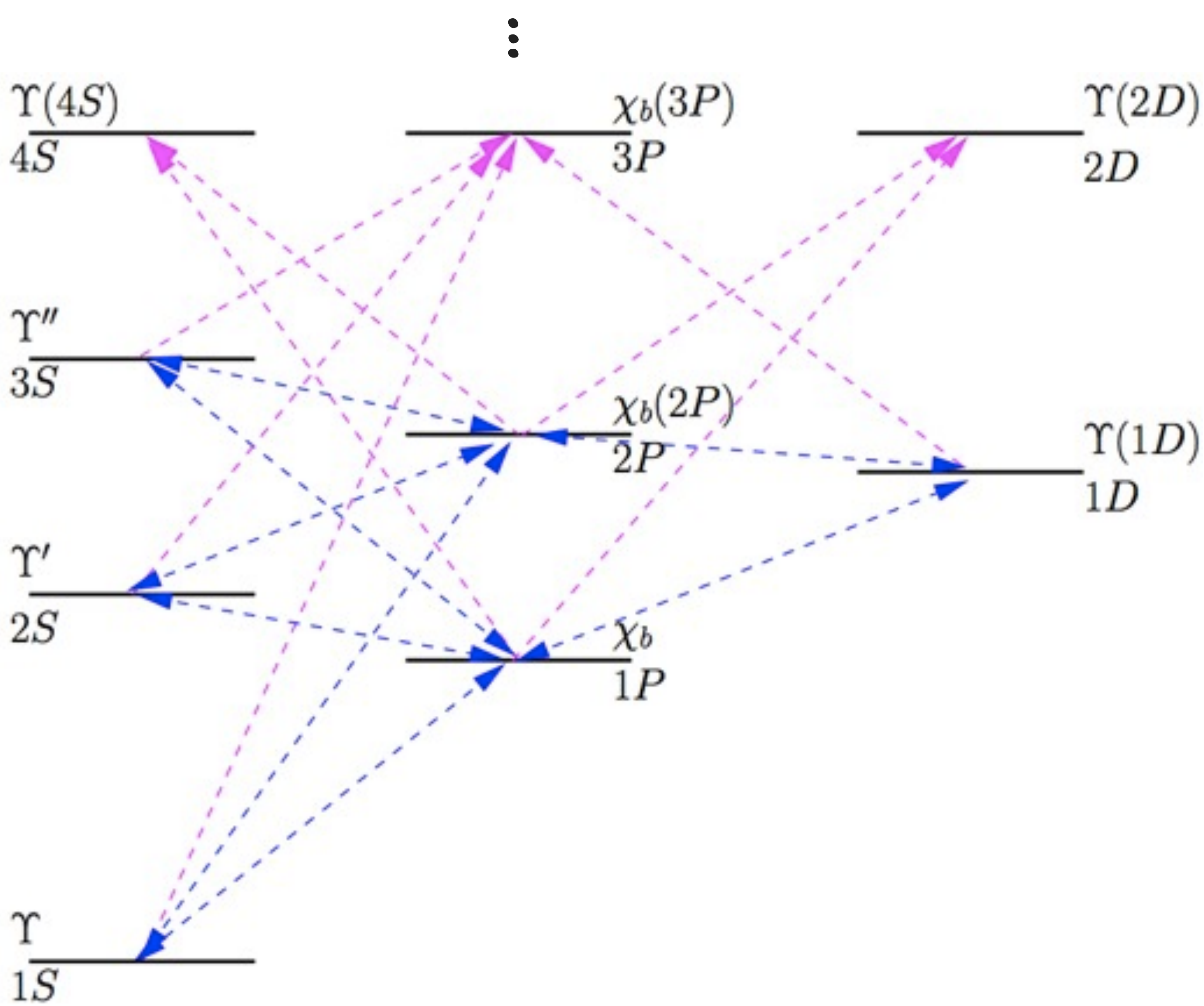
- ... and more precisely, between $b\bar{b}$ states.

why bottomonia? because there are more of them!

(and thereby allow us to bypass "cold nuclear matter" issues?)

Master-equation-based description of bottomonia in a QGP

Simplified (but not simplistic) exploratory model:



Cannot transition back to bound states: ☞ "continuum"

Stable in vacuum.

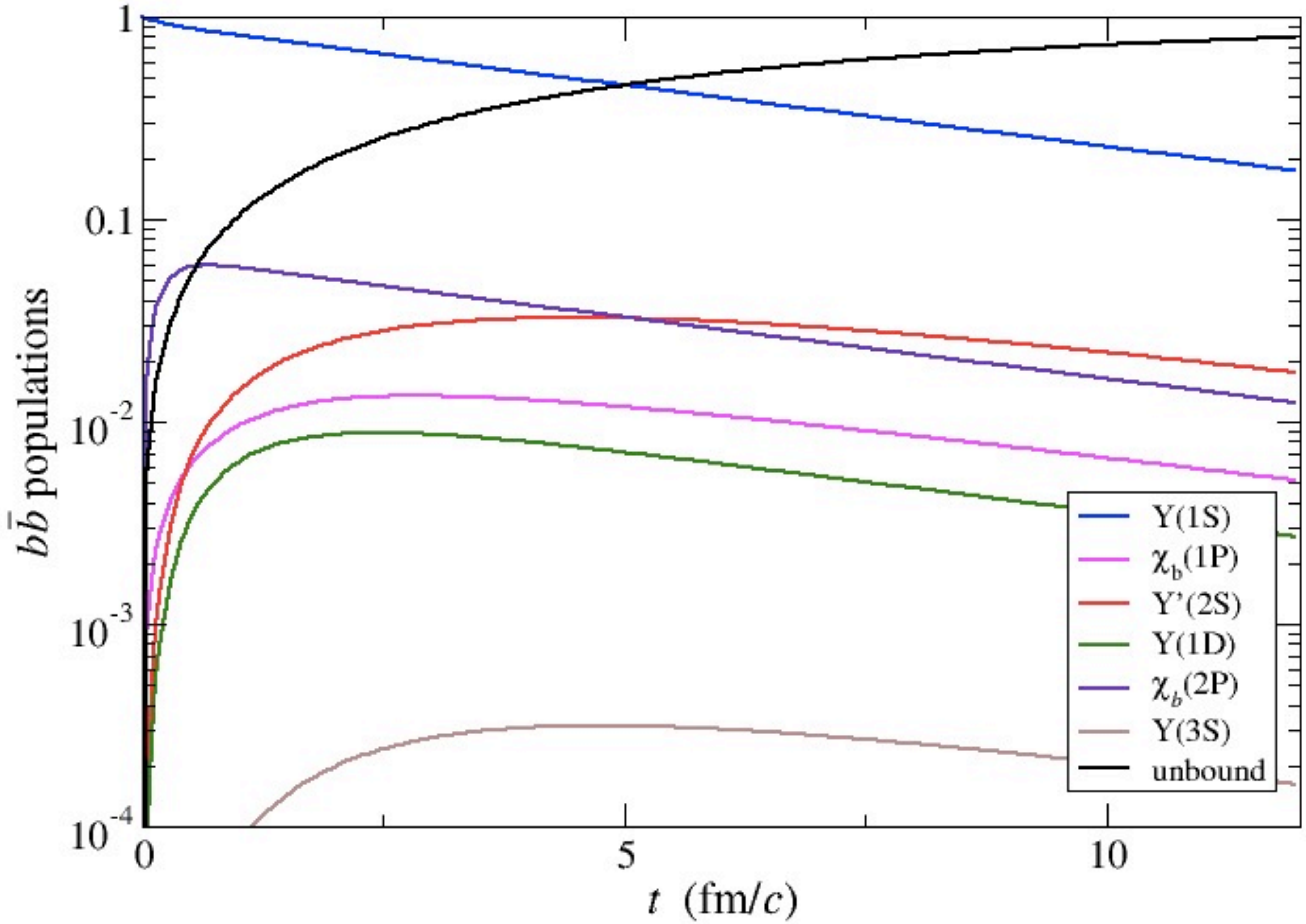
Can absorb or emit bath excitations (2-way transitions)

Some transitions are missing, dissociated states are poorly modeled...

(Static) Bottomonia in a Gaussian bath

...peaked around $5T_c$

Initial condition: only the ground state is populated at t=0.

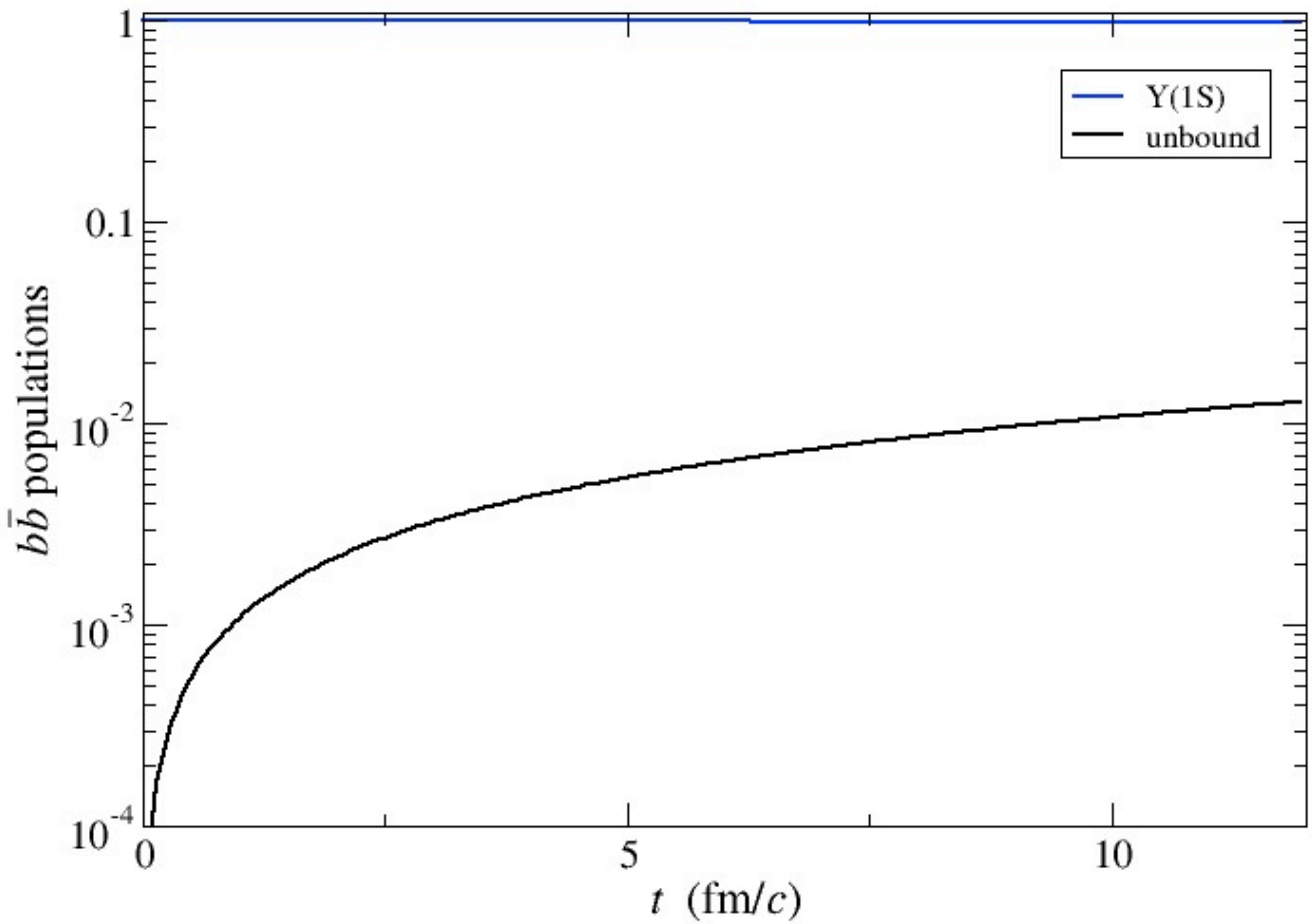


☛ proper energy to induce transitions / dissociation.

(Static) Bottomonia in a Gaussian bath

...peaked around $10T_c$

Initial condition: only the ground state is populated at t=0.

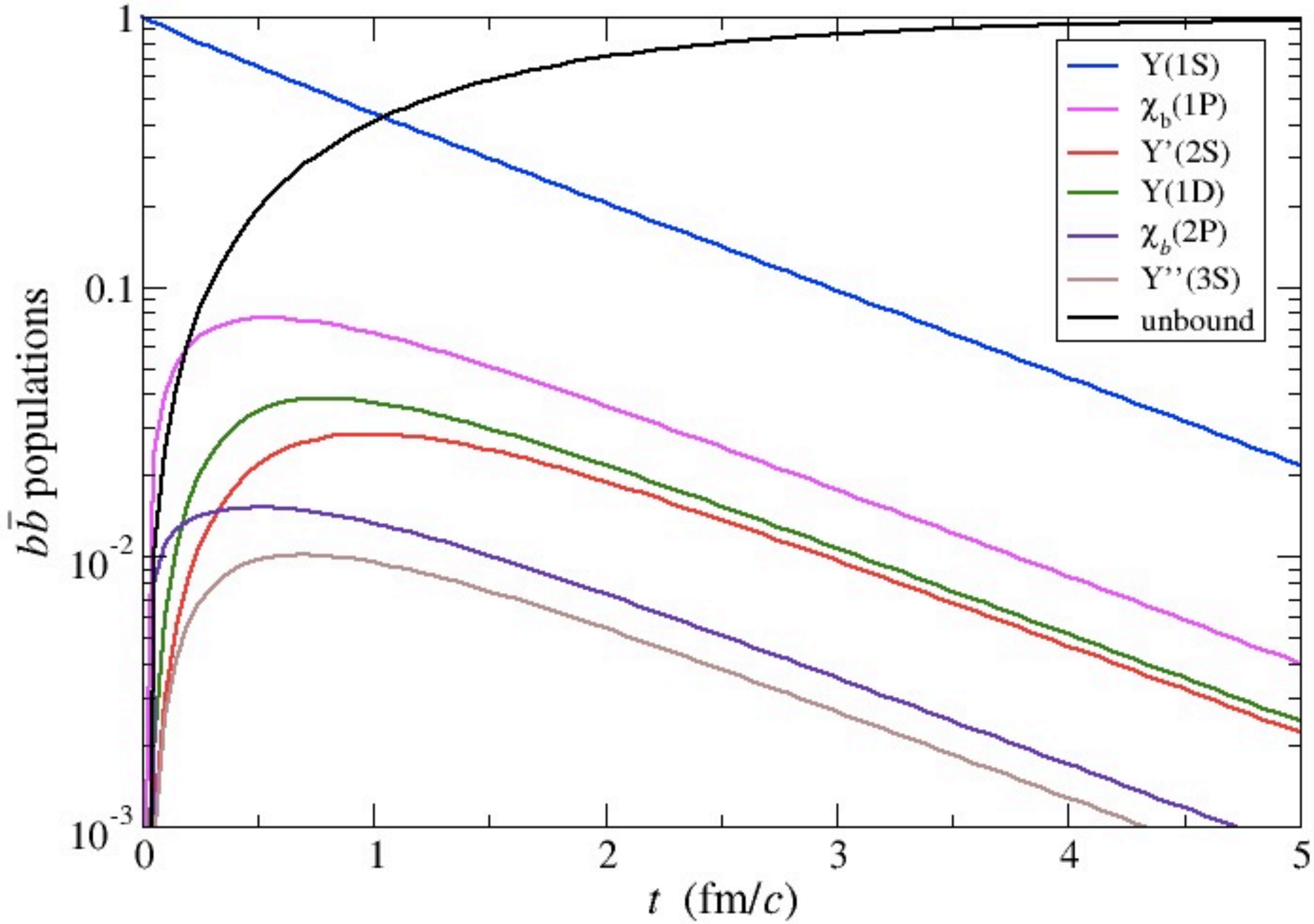


☛ the gluons are too energetic to dissociate the bottomonia.

(Static) Bottomonia in a thermal bath

...at T = $5T_c$

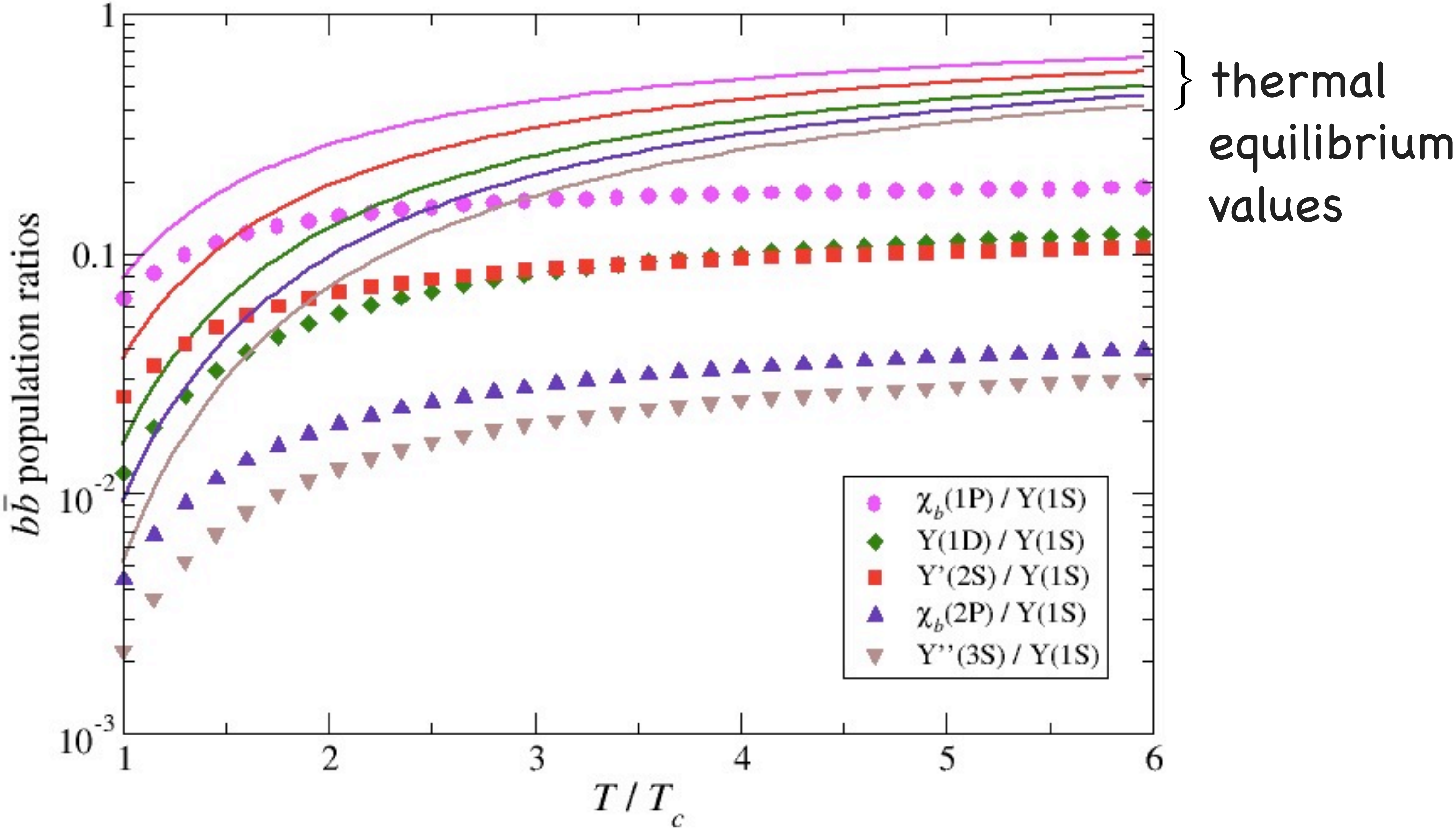
Initial condition: only the ground state is populated at t=0.



☛ After a transient regime, the various bound states evolve together.

(Static) Bottomonia in a thermal bath

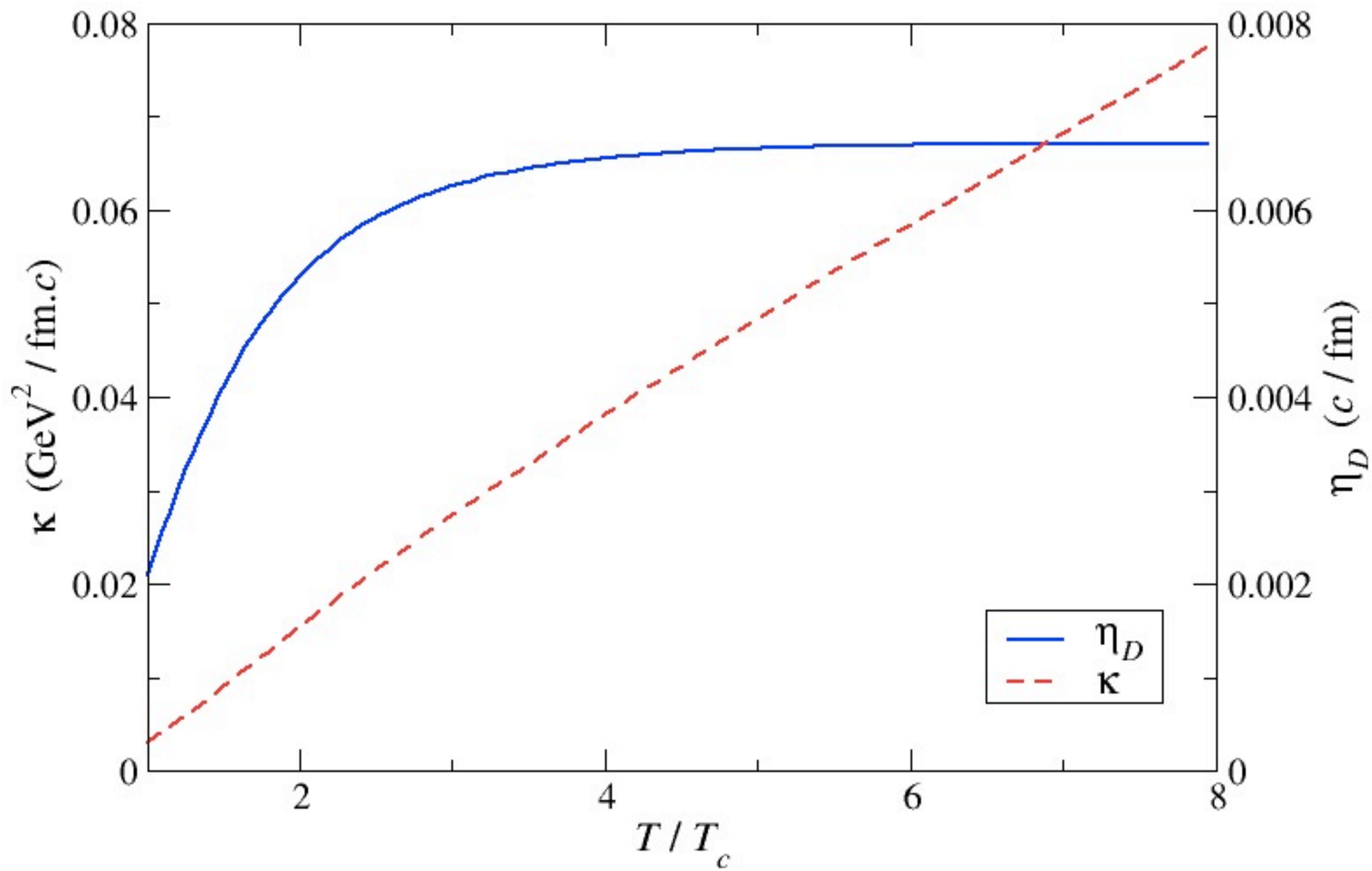
After a transient regime, the various bound states evolve together: (quasi-)equilibrium, the population ratios remain constant



☛ The quasi-equilibrium ratios differ from the statistical model values.

Bottomonia in a thermal bath

After a transient regime, the bound states reach a (quasi-)equilibrium, in which their momentum distribution obeys a Fokker-Planck equation.



But...

Master-equation-based description of bottomonia in a QGP

- After a transient regime, the various bound $b\bar{b}$ states immerged in a QGP evolve together.
  - ☛ differs from the sequential melting picture

- At quasi-equilibrium, the population ratios differ from those found in statistical models

- ...and the bottomonium momentum distribution obeys a Fokker–Planck equation*.

☞ Modeling as a dissipative quantum system is promising!

* at least in the non-relativistic regime

Master-equation-based description of bottomonia in a QGP

- After a transient regime, the various bound $b\bar{b}$ states immerged in a QGP evolve together.
  - ☛ differs from the sequential melting picture

- At quasi-equilibrium, the population ratios differ from those found in statistical models

- ...and the bottomonium momentum distribution obeys a Fokker–Planck equation*.

☞ Modeling as a dissipative quantum system is promising!

BUT... what about the time scales?

Master-equation-based description of bottomonia in a QGP

- After a transient regime, the various bound $b\bar{b}$ states immerged in a QGP evolve together. $\Gamma^{-1} \approx 1.5$ fm/c at $5T_c$ (≈ 8 fm/c at $2T_c$)
  - ☛ differs from the sequential melting picture

- At quasi-equilibrium, the population ratios differ from those found in statistical models

- ...and the bottomonium momentum distribution obeys a Fokker–Planck equation*. $\eta_D^{-1} \approx \mathcal{O}(10^2$ fm/$c)$

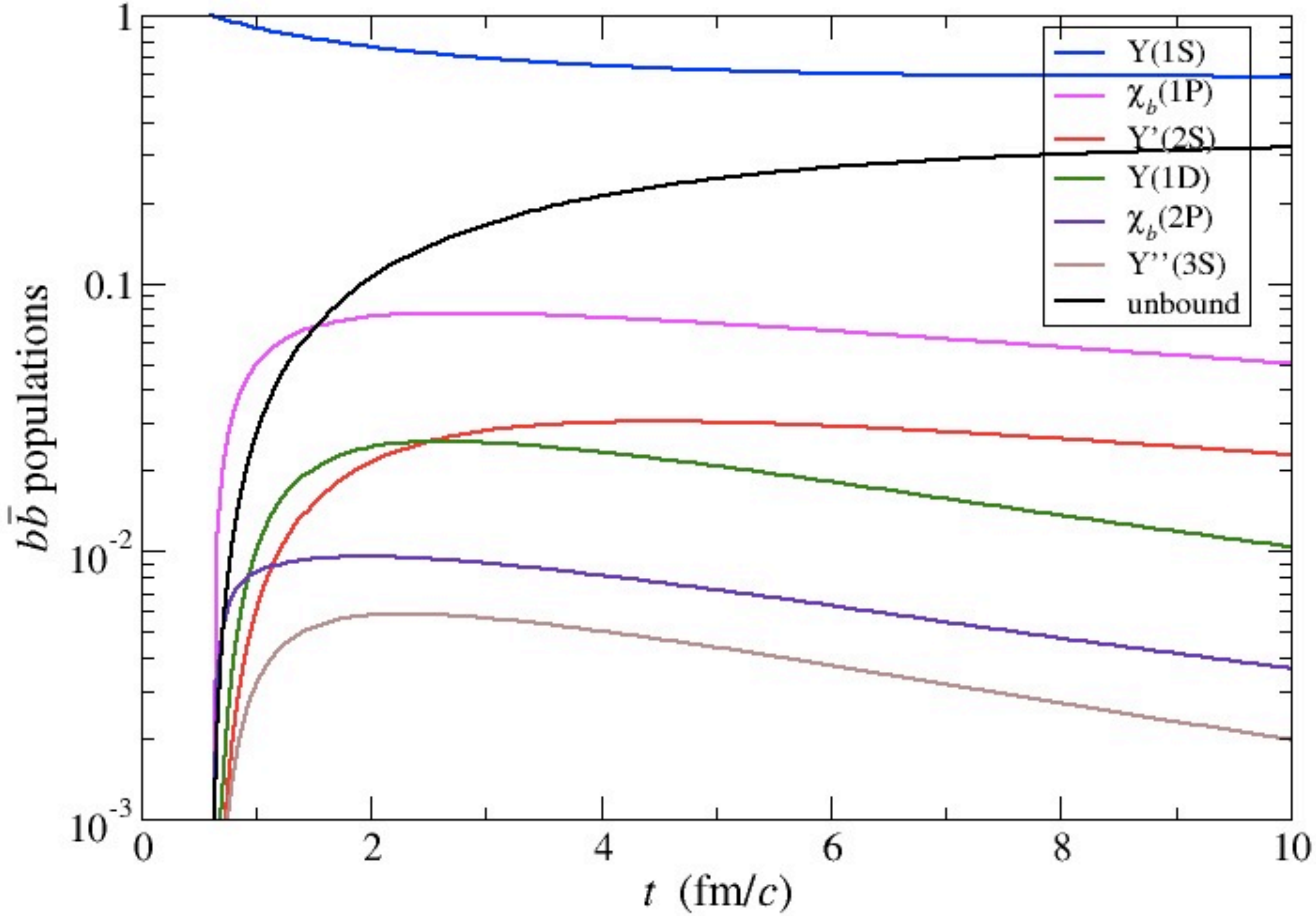
The leading behavior here is the disappearance of the bound states!

☞ Modeling as a dissipative quantum system is promising!

BUT... what about the time scales?

Bottomonia in a QGP with evolving temperature

using the time-dependence of temperature as computed by Shen et al., arXiv:1005.3226



☛ The bound states do not have time to equilibrate with each other!

Heavy quarkonia in a medium as a dissipative quantum system: Master equation approach

- Modeling the real-time dynamics of quarkonia in a (deconfined) medium as those of a dissipative quantum system seems to be viable.

- The master-equation approach hints at possible behaviors
  - regeneration of excited states
  - not enough time to develop Fokker–Planck dynamics(?)

- Need to investigate more realistic microscopic models as well as alternative approaches to dissipative quantum systems

  ☛ next talk by Nirupam Dutta!

and to make contact with existing descriptions.

extra slides

Time-dependence of temperature

At the center of the hydrodynamically expanding fireball created in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV   Shen et al., arXiv:1005.3226