# Statistical description of the initial state and validity of mode-by-mode dynamics

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QUARK MATTER KRAKÓW 2022

Bielefeld University

06.04.2022

CRC-TR 211

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# Statistical analysis of initial state

- Take sample of events $\Phi^{(i)}$ (at fixed $\vec{b}$ in this "poster")
- Average state:

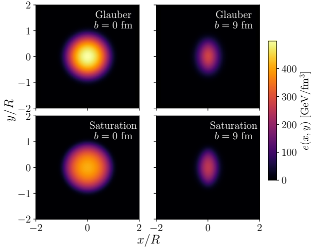
$$\bar{\Psi} \equiv \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} \Phi^{(i)}$$

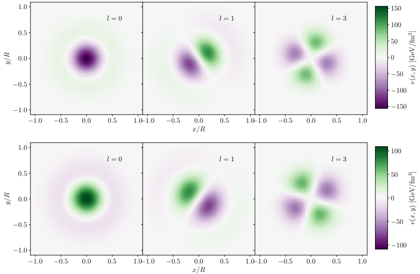
- Each event can be decomposed into an average state and fluctuation modes:

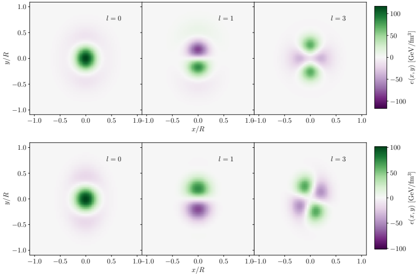
$$\Phi^{(i)} = \bar{\Psi} + \sum_l c_l \Psi_l \quad \text{with} \quad \langle c_l \rangle = 0$$

- Fluctuations in different modes are uncorrelated: $\langle c_l c_l' \rangle = \delta_{l,l'}$

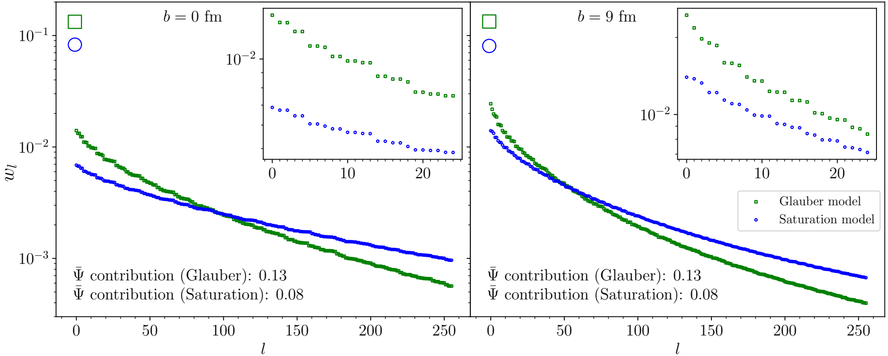
# Average initial states and modes



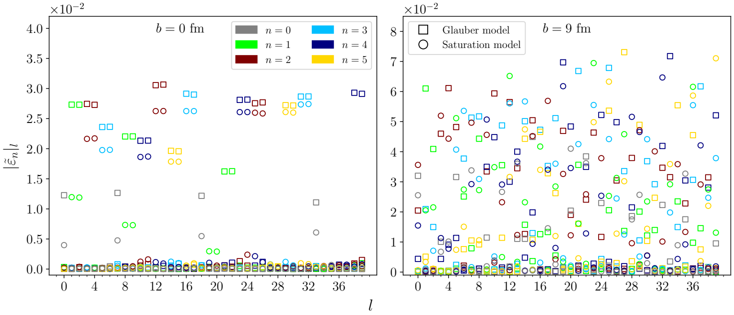
# Typical relative weight of modes




$$w_l \equiv \frac{||\Psi_l||}{\sum_l ||\Psi_l|| + ||\bar{\Psi}||}, \quad \bar{w} \equiv \frac{||\bar{\Psi}||}{\sum_l ||\Psi_l|| + ||\bar{\Psi}||}$$

- $\approx 10\%$ contribution from $\bar{\Psi}$
- Singlets and doublets
- $w_l$ quantifies the importance of $\Psi_l$

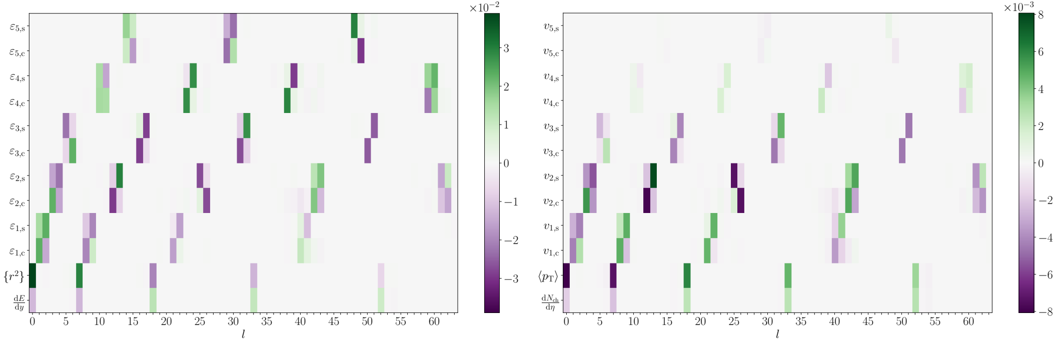
# Mode energy and eccentricities




$$\tilde{\varepsilon}_n e^{in\Phi_n} \equiv -\frac{\int r^n e^{in\theta}\Psi_l(r,\theta)\, r\mathrm{d}r\mathrm{d}\theta}{\int r^n \bar{\Psi}(r,\theta)\, r\mathrm{d}r\mathrm{d}\theta} \quad \text{for } n \neq 1$$

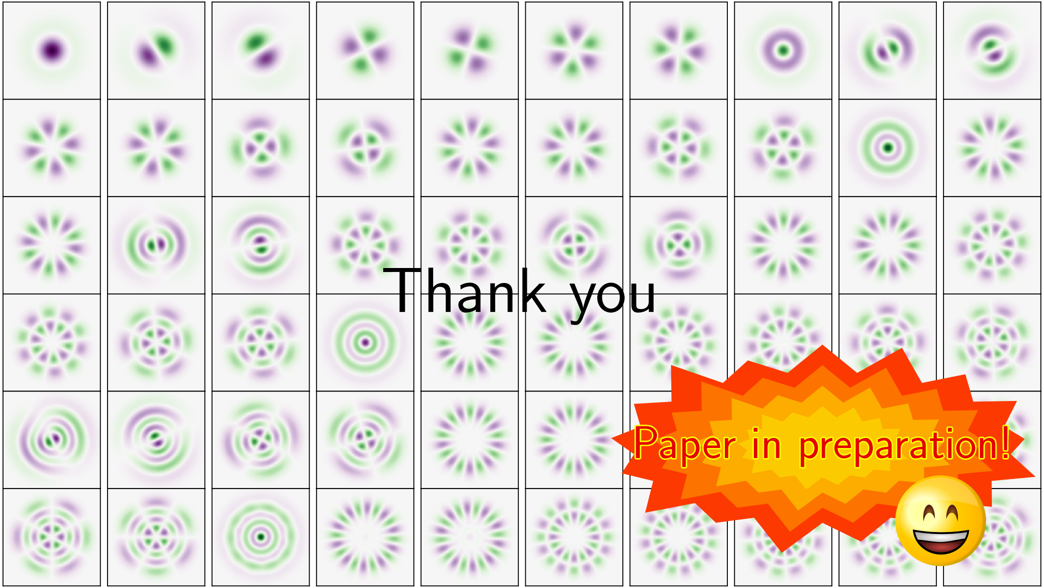
- Singlet and doublet structure $b = 0$ fm (rotational symmetry)
- Radial modes contain energy ($n = 0$)
- $b = 9$ fm: multiple $\tilde{\varepsilon}_n$ for each $l$

# Mode-by-mode perturbation and dynamical evolution ($b = 0$)




$\xrightarrow{\hspace{1cm}}$
KøMPøST + MUSIC

- Mode-by-mode perturbation of $\bar{\Psi}$
  $\rightarrow$ linear response of initial state
- Dynamical response in final state
  $\rightarrow$ viscous damping for large $n$




# Thank you

# Backup

# (Non-)linear response theory

- Write observables as

$$O_\alpha = \bar{O}_\alpha + \frac{\partial O_\alpha}{\partial c_l}\bigg|_{\bar{\Psi}} c_l + \frac{1}{2}\frac{\partial^2 O_\alpha}{\partial c_l \partial c_{l'}}\bigg|_{\bar{\Psi}} c_l c_{l'} + \mathcal{O}(c_l^3)$$
$$\equiv \bar{O}_\alpha + L_{\alpha,l} c_l + \frac{1}{2} Q_{\alpha,ll'} c_l c_{l'} + \mathcal{O}(c_l^3).$$

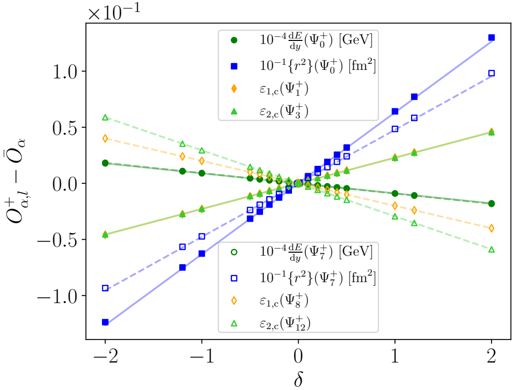
- Define states

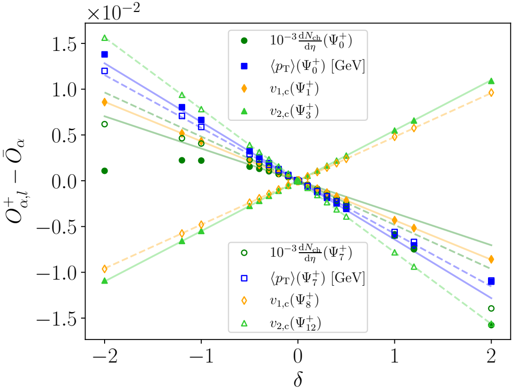
$$\Psi_l^+ \equiv \bar{\Psi} + \delta\Psi_l, \quad \Psi_l^- \equiv \bar{\Psi} - \delta\Psi_l$$

- Compute the linear and quadratic response:

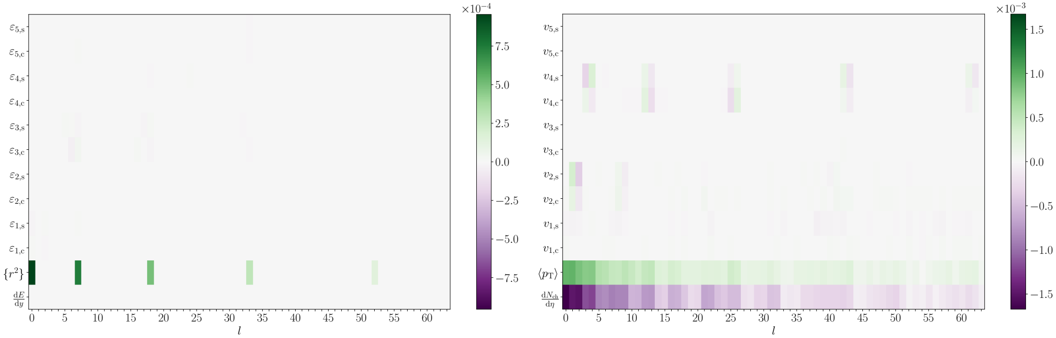
$$L_{\alpha,l} = \frac{O_{\alpha,l}^+ - O_{\alpha,l}^-}{2\delta}, \quad Q_{\alpha,ll} = \frac{O_{\alpha,l}^+ + O_{\alpha,l}^- - 2\bar{O}_\alpha}{\delta^2}$$

# Linearity check



# Non-linear response ($b = 0$ fm)

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