Constraining dissipative corrections to particle distributions at freeze-out from anisotropic flow

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Sudden Freeze-Out Approximation

- Matter produced in heavy-ion collisions follows fluid dynamics (continuous medium)
- The detectors observe particles, created matter is no longer a continuous medium

→ **How does this transition proceed?**

- Simple, yet promising ansatz:

Sudden Freeze-Out Approximation

- Sharp transition between fluid and particles:
  Define a hyper-surface Σ on which the transition is expected to take place;
  at each point of Σ free streaming particles are emitted according to the thermal distributions in the rest frame of the fluid

Cooper-Frye Formula

Momentum spectrum for a given particle:

$$E_{\vec p}\frac{d^3N}{d^3\vec p}=\frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{p\cdot u(x)}{T}\right)p^\mu d^3\sigma_\mu(x)$$

- f is the single particle occupation factor (e.g. Bose-Einstein-/Fermi-Dirac-distribution)
- Here: **Classical ideal fluid** $(f\to f_0)$

$$f_0\left(\frac{p\cdot u(x)}{T}\right)\propto e^{-\frac{p\cdot u(x)}{T}}$$

→ Equilibrium thermal distribution f_0 (Maxwell-Boltzmann-distribution)

- **Remark:** (Initial) fluctuations are ignored

Decoupling from an ideal fluid

- Computation of the Cooper-Frye integral via method of steepest descent: **[1]**
- **Results:**

Two „kinds“ of particles: **slow and fast particles** depending on the tranverse momentum p_t

$$p_t<m_t\bar u_{max}\quad slow\ particles$$
$$p_t>m_t\bar u_{max}\quad fast\ particles$$

with $\bar u_{max}$ the maximum value for $|\boldsymbol{u}_t|$ with a fixed rapidity and azimuth

- Stronger constraints on p_t are needed to ensure validity of the approximation

From an ideal fluid to a dissipative fluid

- Four-velocity $u_\mu(x)$ is changed

→ $u_\mu(x)$ has to be a solution of the relativistic Navier-Stokes or second-order equations

→ This fact can be neglected here, because the focus is on corrections arising at freeze-out

- f_0 in the Cooper-Frye formula receives correction terms, depending on the dissipative effects: **[2]**

→ First-order corrections δf_1: shear and bulk viscosity

$$f_0\to f_0+\delta f_{1shear}+\delta f_{1bulk}$$
$$\to\ \delta f_{1shear}=C_{shear}(p\cdot u(x))\,\pi^{\mu\nu}(x)\,p_\mu p_\nu f_0(p\cdot u(x))$$
$$\to\ \delta f_{1bulk}=C_{bulk}(p\cdot u(x),p^2)\,\Pi(x)\,f_0(p\cdot u(x))$$

→ Second-order corrections δf_2: only partialy known

- C_{shear} and C_{bulk}: not really known → Different options for possible models (e.g. Grad prescription)

Decoupling from a dissipative fluid

Slow Particles

- Emitted from point x, where $u_\mu=\frac{p_\mu}{m}$
- The shear tensor must fullfil the Landau relation: $\pi^{\mu\nu}(x)u_\mu(x)=0$

→ $\delta f_{1shear}=0$, since for slow particles: $p_\mu\propto u_\mu$

- The factor C_{bulk} is a function of $p^2=m^2$

→ The contribution of δf_{1bulk} is identical for particles with the same four-velocity u_μ

- **Conclusion:**

Same result as in the ideal case **[3]**

$$E_{\vec p}\frac{d^3N}{d^3\vec p}=c(m)F\left(\frac{p_t}{m},y,\Phi\right)$$
$$mass\ ordering\ of\ v_n(p_t,y)$$

with modified c and F compared to the results for the ideal fluid **[1]**

Fast Particles

- Idea:

Fast particles, emitted in a given direction, all come from the same saddle point, where the fluid velocity reaches at its maximum u_{max}

- At the saddle point:

$$p\cdot u(x)=m_t\sqrt{1+u_{max}(\varphi)^2}-p_t u_{max}(\varphi)$$

→ Governs the momentum spectrum:

$$E_{\vec p}\frac{d^3N}{d^3\vec p}\propto e^{\frac{p_t u_{max}-m_t\sqrt{1+u_{max}^2}}{T}}$$

- u_{max} dependence on the azimuth:

$$u_{max}(\varphi)=\bar u_{max}\left(1+\sum_{n=1}2V_n\cos(n\varphi)\right)$$

→ Yield the "ideal" flow coefficients v_n

$$v_2(p_t)^{ideal}=V_2 I(p_t)$$
$$v_3(p_t)^{ideal}=V_3 I(p_t)+o(V_1V_2)$$
$$v_4(p_t)^{ideal}=V_4 I(p_t)+V_2^2\frac{I(p_t)^2}{2}$$
$$v_5(p_t)^{ideal}=V_2V_3 I(p_t)+o(V_5)$$

$$I(p_t)=\frac{\bar u_{max}}{T}[p_t-m_t\bar v_{max}]\qquad \bar v_{max}=\frac{\bar u_{max}}{\sqrt{1+\bar u_{max}^2}}$$

Decoupling from a dissipative fluid

Fast particles

- Adding the first-order correction terms:

$$E_{\vec p}\frac{d^3N}{d^3\vec p}\propto e^{\frac{p_t u_{max}-m_t\sqrt{1+u_{max}^2}}{T}}(1+\delta f_{1bulk}+\delta f_{1shear})$$

- An **example** for the correction terms:
  - δf_{1bulk} will be neglected for simplicity
  - δf_{1shear} remains

→ Only the $\pi^{rr}(x)$-term contributes: $\delta f_{1shear}=C_{shear}\eta[p_t-m_t v_{max}]^2\langle\nabla^r u^r\rangle$

Momentum spectrum

$$E_{\vec p}\frac{d^3N}{d^3\vec p}\propto e^{\frac{p_t u_{max}-m_t\sqrt{1+u_{max}^2}}{T}}(1+C_{shear}\eta[p_t-m_t v_{max}]^2\langle\nabla^r u^r\rangle)$$

Flow coefficients [3]

$$v_2(p_t)=V_2[I(p_t)-D(p_t)]$$
$$v_3(p_t)=V_3[I(p_t)-D(p_t)]+o(V_1V_2)$$
$$v_4(p_t)=V_4[I(p_t)-D(p_t)]+V_2^2\left[\frac{I(p_t)^2}{2}-I(p_t)D(p_t)\right]$$
$$v_5(p_t)=V_2V_3[I(p_t)-D(p_t)]+o(V_5)$$

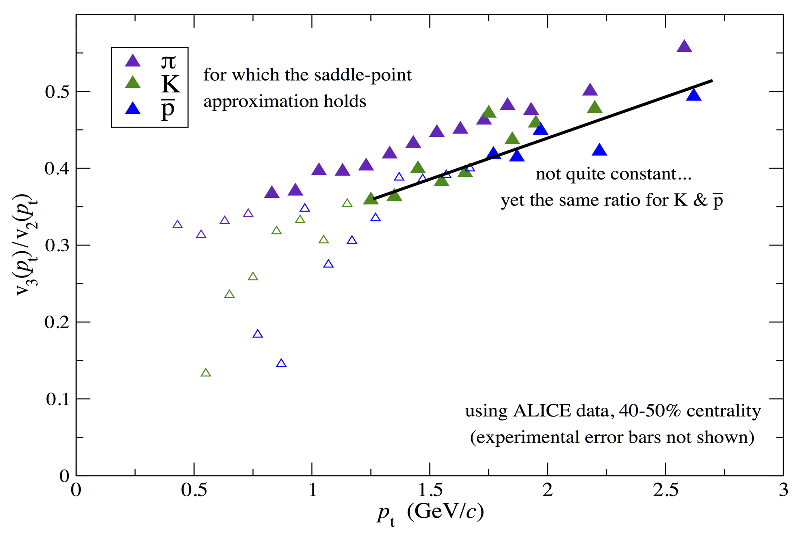
$$D(p_t)=\frac{m_t\bar v_{max}}{p_t-m_t\bar v_{max}}\frac{2}{1+\bar u_{max}^2}h\left(\frac{p_t-m_t\bar v_{max}}{T}\right)$$
$$h(\xi)=\frac{g(\xi)}{1+g(\xi)}$$
$$g(\xi)=\xi^2\eta C_{shear}(x_{s.p.})\langle\nabla^r u^r\rangle(x_{s.p.})$$

→ $D(p_t)$ comes from the dissipative effects

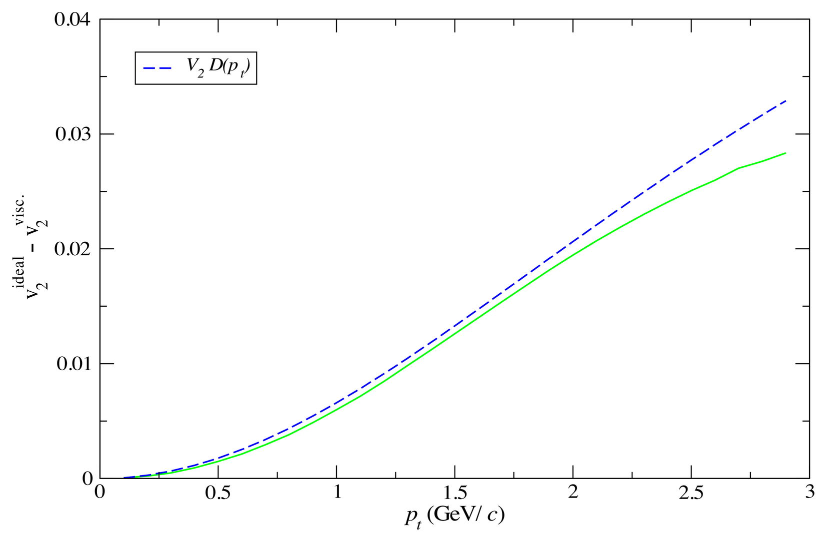
Dissipative effects [3]

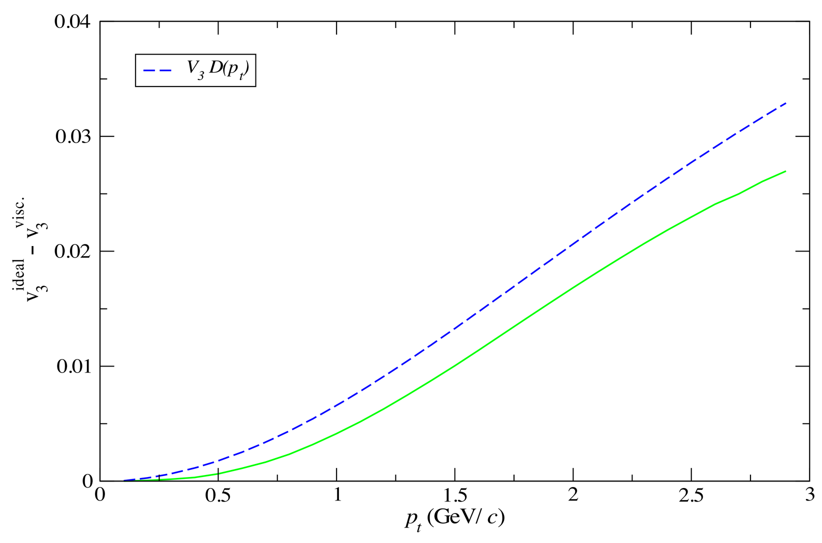
- $\frac{v_3(p_t)}{v_2(p_t)}=\frac{V_3}{V_2}$

The ratio between v_2 and v_3 should be a constant, in fact, if $V_2=V_3$ and all other $V_n=0$, then v_2 and v_3 should not be any different and the ratio should be equal to one



- $v_2(p_t)^{ideal}-v_2(p_t)^{visc.}=V_2D(p_t)$ $\qquad v_3(p_t)^{ideal}-v_3(p_t)^{visc.}=V_3D(p_t)+o(V_1V_2)$





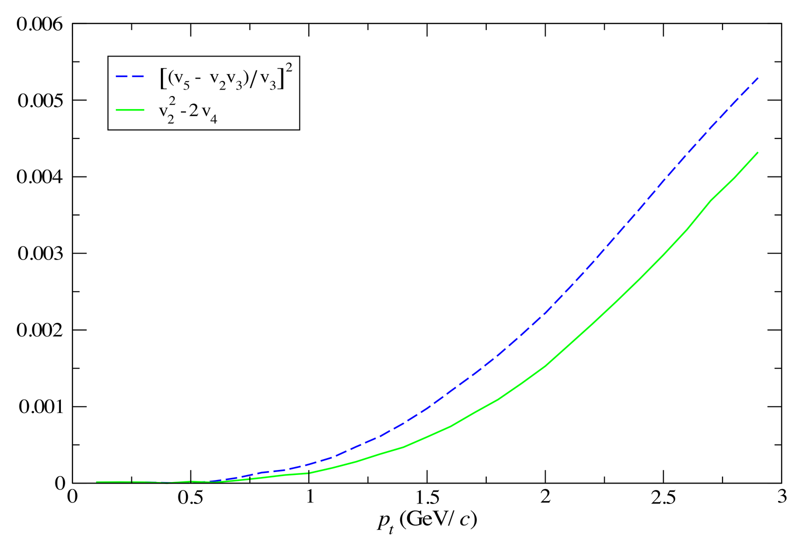
For $\bar u_{max}$ not too large ($\bar u_{max}<<1$), the analytic solution $D(p_t)$ (blue) can be compared to the disspative correction calculated with a "blast wave model" ansatz (green)

- $\frac{(v_5(p_t)-v_2(p_t)v_3(p_t))^2}{v_3(p_t)^2}=V_2^2D(p_t)^2$

$$2v_4(p_t)-v_2(p_t)^2=V_2^2D(p_t)^2$$

Two independent relations from which the viscous correction could be extracted

→ Further results to compare with the analytic calculation $D(p_t)$



Summary

- Slow particles: Viscous corrections do not change the scaling laws
- Fast particles: Flow coefficients gain a viscous correction term
- Flow coefficients: New relations between different flow coefficients are found

Outlook

- Other relations possible
- Other freeze-out ansätze

References

[1] Nicolas Borghini, Jean-Yves Ollitrault; Physics Letters B 642 (2006) 227-231

[2] Derek Teaney; Physical Review C 68, 034913 (2003)

[3] Christian Lang, Nicolas Borghini; arXiv:1312.7763