Statistical description of the initial state and mode-by-mode dynamics

Initial Stages 2023

Nicolas Borghini⁺, Marc Borrell, Nina Feld⁺, Hendrik Roch*, Sören Schlichting⁺, Clemens Werthmann°

⁺Bielefeld University, Germany / *Frankfurt Institute for Advanced Studies, Germany / °University of Wroclaw, Poland

* roch@fias.uni-frankfurt.de

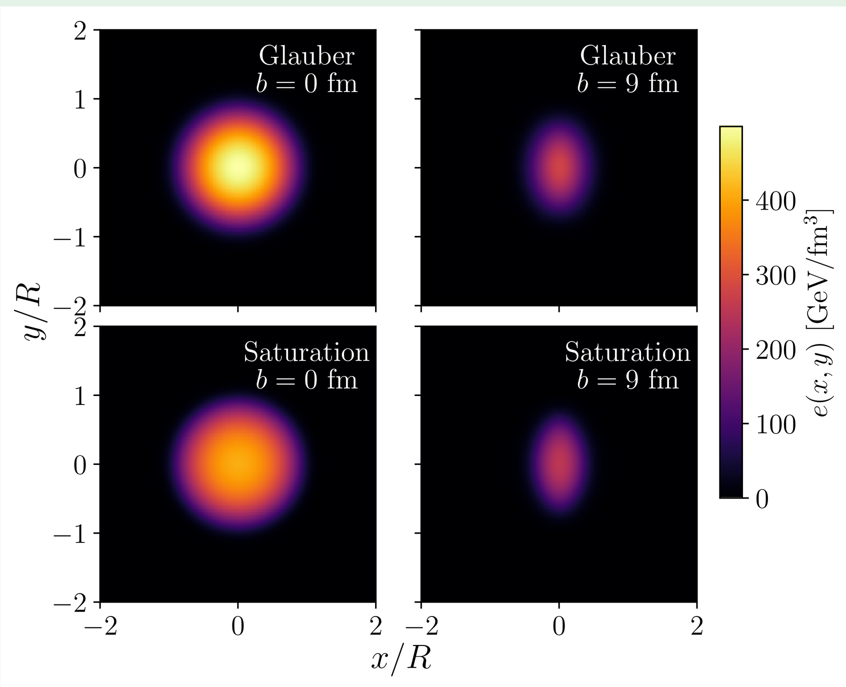
1. Introduction

- Systematic characterization of an ensemble of initial density profiles $\{\Phi^{(i)}\}$ using an average state $\bar{\Psi}$ and statistically uncorrelated modes $\{\Psi_l\}$ representing event-by-event fluctuations [1].
- We quantify types and probabilities of fluctuations and study the impact on initial state quantities.
- Using KøMPøST and MUSIC we investigate the influence of various modes on final-state bulk observables and their correlations.

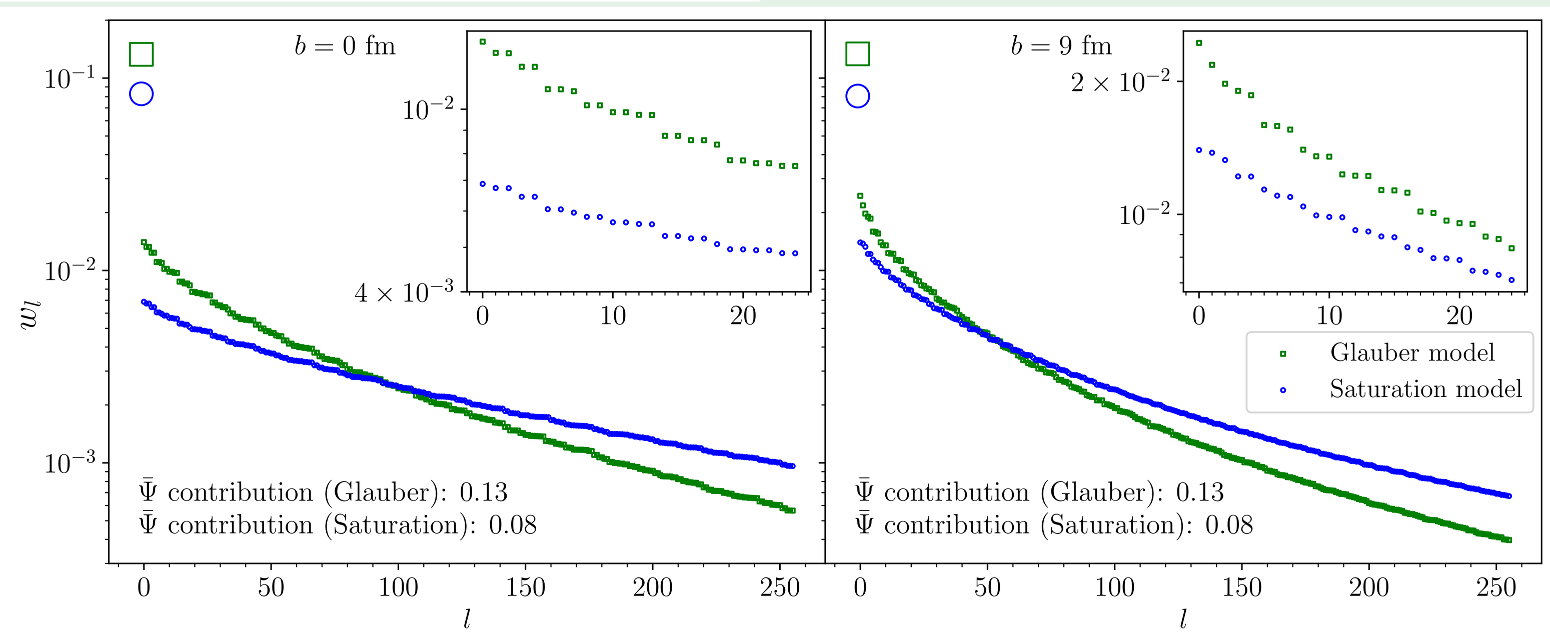
2. Theoretical Formulation

- Decompose events as $\Phi^{(i)}(\mathbf{x}) = \bar{\Psi}(\mathbf{x}) + \sum_l c_l^{(i)} \Psi_l(\mathbf{x})$ with $\langle c_l \rangle = 0$, $\langle c_l c_{l'} \rangle = \delta_{ll'}$
- Average state: $\bar{\Psi}(\mathbf{x}) \equiv \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} \Phi^{(i)}$
- Fluctuation modes are eigenvectors of the density matrix: $\rho \equiv \frac{1}{N_{\rm ev}} \sum_i \Phi^{(i)} \Phi^{(i)\mathsf{T}} - \bar{\Psi}\bar{\Psi}^{\mathsf{T}}$
- Eigenvalues λ_l of ρ represent the strength of the fluctuation modes.
- Apply this framework to two different initial-state models (Pb-Pb @ 5.02 TeV):
  - MC-Glauber (nucleons) / $k_{\rm T}$-factorization-based Saturation
  - fixed impact parameter, 2D

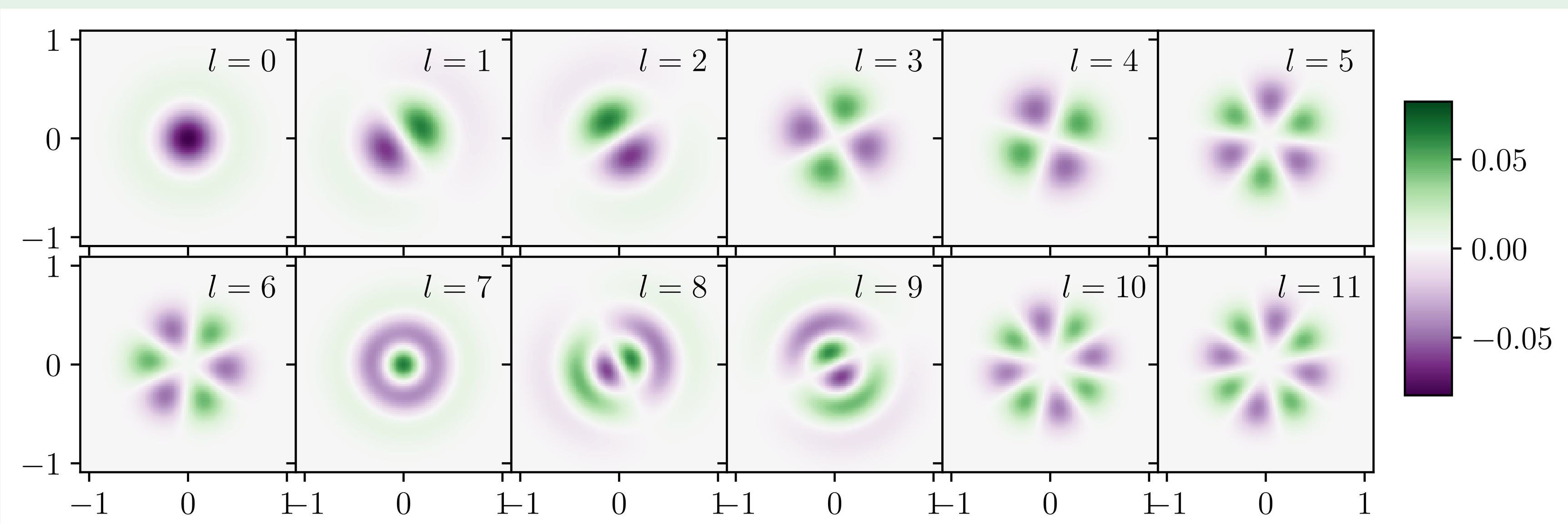
3. Average States, Eigenvalues & Modes

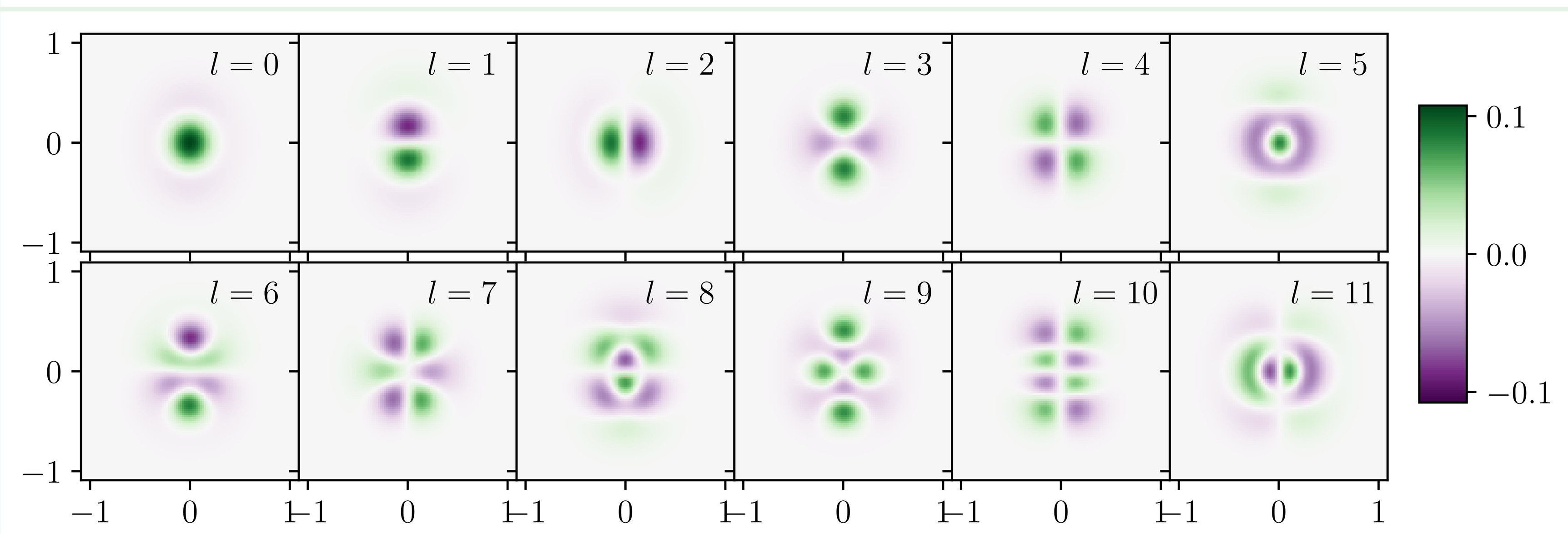


- Rotationally symmetric $\bar{\Psi}$ for zero impact parameter.
- Relative importance of $\bar{\Psi}$: $\bar{w} \equiv \frac{||\bar{\Psi}||}{\sum_l \sqrt{\lambda_l} + ||\bar{\Psi}||}$
- Relative importance of Ψ_l: $w_l \equiv \frac{\sqrt{\lambda_l}}{\sum_l \sqrt{\lambda_l} + ||\bar{\Psi}||}$

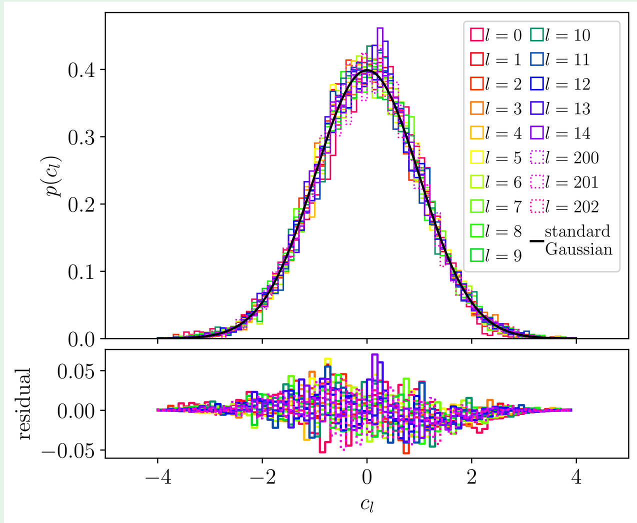


- Exemplary most important modes in the Glauber model for $b = 0$ (top) and $b = 9$ fm (bottom)





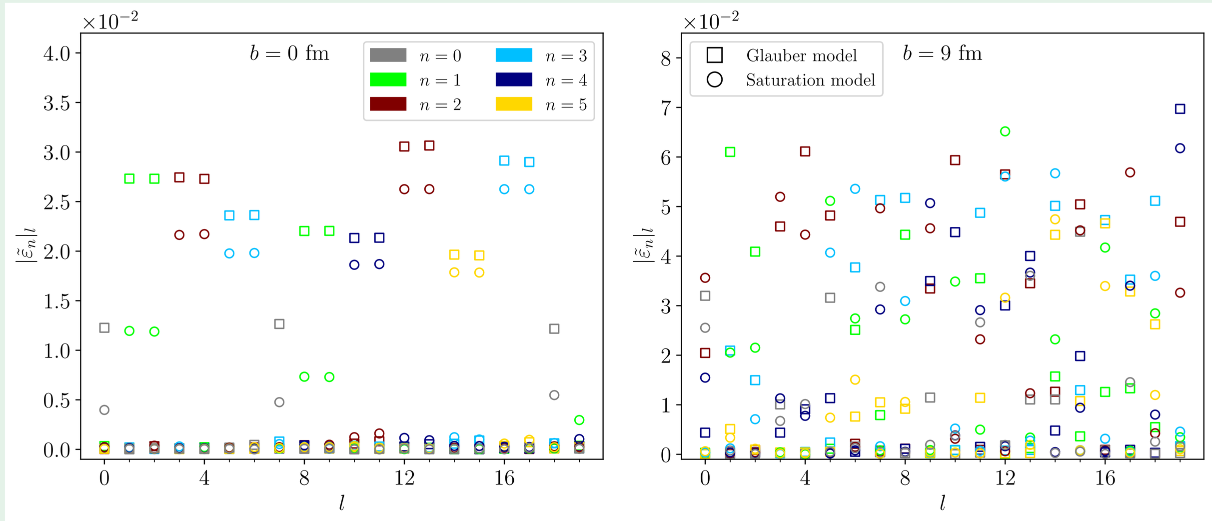
4. c_l Distributions



- Expansion coefficients c_l follow an almost Gaussian distribution (here Glauber model $b = 0$).
- Deviations from Gaussianity are larger at finite impact parameter and for the modes with large l (not shown).

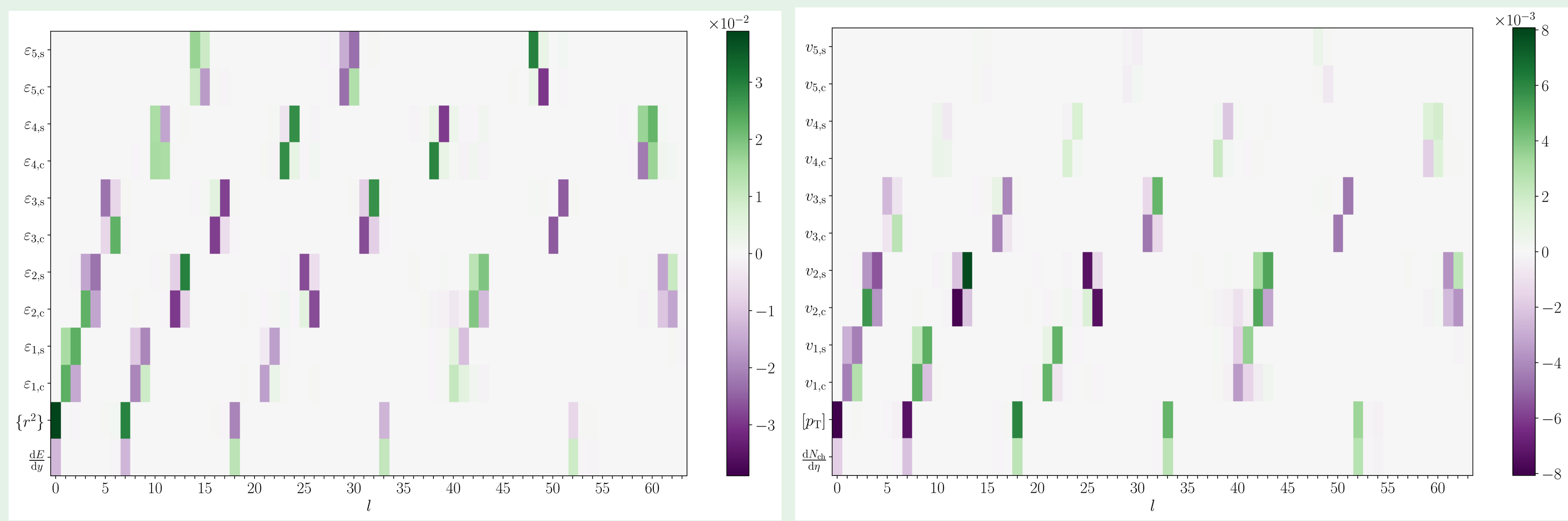
5. Eccentricities

- $\tilde{\varepsilon}_n(\Psi_l) \equiv -\frac{\int r^n e^{in\theta} \Psi_l(r,\theta)\, r\, {\rm d}r\, {\rm d}\theta}{\int r^n \bar{\Psi}(r,\theta)\, r\, {\rm d}r\, {\rm d}\theta}$ for $n \geq 2$, r^3-weight for $n = 1$.
- At $b = 0$, we find radial modes that contribute to the energy ($n = 0$) and degenerate doublets with a single sizable ε_n.
- At $b = 9$ fm a mode can have multiple eccentricities.



6. (Non-)linear Response

- Expand observables: $O(\Phi) = O(\bar{\Psi}) + \sum_l L_l c_l + \frac{1}{2} \sum_{l,l'} Q_{ll'} c_l c_{l'} + \mathcal{O}(c_l^3)$, where $\Phi = \bar{\Psi} + \sum_l c_l \Psi_l$ and compute response coefficients L_l, $Q_{ll'}$ numerically.
- Example: linear-response coefficients at $b = 0$ (Glauber model).
  - Initial-state quantities(left):
    - ⋆ Radial modes affect size and energy; eccentricities (Re and Im ε_n) for mode pairs.
  - Final-state observables (right):
    - ⋆ Radial modes affect average $p_{\rm T}$ and ${\rm d}N_{\rm ch}/{\rm d}\eta$. Modes with ε_n lead to anisotropic flow v_n.
    - ⋆ Viscous damping of the higher harmonics is visible.



- Quadratic response is in general small compared to linear response, the most sizable contributions are for ${\rm d}N_{\rm ch}/{\rm d}\eta$, $[p_{\rm T}]$ and $\langle r^2 \rangle$.

7. Conclusion & Outlook

- Found an optimal basis with uncorrelated fluctuation modes on top of an average state to decompose the initial-state profiles.
- Future directions:
  - Centrality dependent study with SMASH as hadronic afterburner.
  - Decomposing 3D initial conditions and include other initial state models for comparison.

References & Acknowledgements

[1] Nicolas Borghini, Marc Borrell, Nina Feld, Hendrik Roch, Sören Schlichting, and Clemens Werthmann. Statistical analysis of initial-state and final-state response in heavy-ion collisions. *Phys. Rev. C*, 107(3):034905, 2023.

We acknowledge support by the Deutsche Forschungsgemeinschaft (DFG) through the grant CRC-TR 211 "Strong-interaction matter under extreme conditions". Computational resources have been provided by the Paderborn Center for Parallel Computing (PC^2).