Spectral function of scalar mesons at finite T and μ in a large-N approximation

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- Motivation
- Linear σ-model in the leading order large N approximation
- Trajectory of the σ pole and its relation to the shape of the spectral function
- Introduction of fermions, the $\boldsymbol{\mu - T}$ phase diagram

σ particle in matter

- σ is the quantum fluctuation of the amplitude of the chiral OP $\langle \bar{q}q \rangle$
- any change in the ground state is reflected upon the properties of σ
- through $\sigma \to 2\pi$ decay σ is strongly coupled with the pions

$T = 0$, $n_B = 0$ σ shows up as a broad resonance
in the $I = J = 0$ channel of the $\pi - \pi$ scattering

the existence of σ receives increasing confidence at mass 500-600 MeV

$T \neq 0$, $n_B \neq 0$ m_σ decreases during chiral symmetry restoration
the available phase space for the $\sigma \to 2\pi$ decay is squeezing
threshold enhancement in the spectral function is produced
chance to see σ as a sharp resonance in the matter

T. Hatsuda, T. Kunihiro Phys. Rev. Lett. **55**, 158 (1985), Phys. Rev. **D57**, R6 (1998),
Phys. Rev. Lett. **82**, 2840 (1999)

Large N (leading order) approach to the linear σ model

$O(N)$ symmetric theory with explicit symmetry breaking external field h:

$$L = \frac{1}{2}[(\partial\vec{\phi})^2 - m^2\vec{\phi}^2] - \frac{\lambda}{24N}[\vec{\phi}^2]^2 + \sqrt{N}h\phi^1$$

In the broken symmetry phase: $\vec{\phi} \to (\sqrt{N}\Phi(T) + \phi^1, \phi^i)$

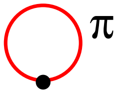
for $\sigma - \pi$ system: $N = 4$, $\Phi(0) = f_\pi/2 \quad f_\pi = 93$ MeV

h determines the pion mass: $m_\pi^2(T) = h/\Phi(T)$

Goal: to determine at finite temperature the trajectory of the σ pole and its relation to the structure of the spectral function by taking advantage of the large N expansion

+ makes strongly self-coupled theory treatable
+ the approach is insensitive to the choice of the renormalisation point
+ leads to a 2nd order chiral transition for $h = 0$ and provides correct critical description near $T_c = 160$ MeV

Quantities of interest to leading order in N

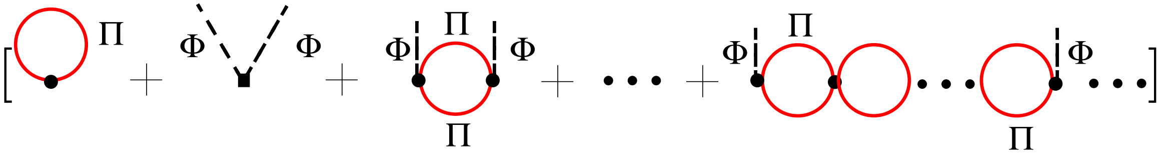
Equation of state: $m^2 + \frac{\lambda}{6}\Phi^2(T) +$  $- \frac{h}{\Phi} = 0$

renormalised with $\frac{m^2}{\lambda} + \frac{\Lambda^2}{96\pi^2} = \frac{m_R^2}{\lambda_R}$ $\frac{1}{\lambda} + \frac{1}{96\pi^2}\ln\frac{e\Lambda^2}{M_0^2} = \frac{1}{\lambda_R}$

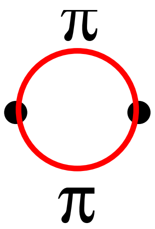
implies $G_\pi^{-1}(p) = p^2 - \frac{h}{\Phi(T)}$ Goldstone theorem when $h \to 0$.

σ pole: solution of $G_\sigma^{-1}(p) = 0$ on the complex plane

$$G_\sigma^{-1}(p) = p^2 - m^2 - [$$



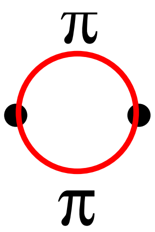
$$= p^2 - \frac{h}{\Phi(T)} - \frac{\lambda_R\Phi^2(T)/3}{1 - \lambda_R b_R(p)/3}$$

$b(p_0) =$  $= b_0(p_0) + b_T(p_0)$ originally defined on the upper halfplane.

Spectral function: $\rho_\sigma(p_0, \mathbf{p} = 0, T) = -\frac{1}{\pi}\lim_{\varepsilon\to+0}\mathrm{Im}G_\sigma(p_0 + i\varepsilon, \mathbf{0}, T)$

A. Patkós, Zs. Sz., P. Szépfalusy Phys. Lett. **B537** (2002) 77,
Phys. Rev. **D66** (2002) 116004

Analytical continuation to the 2$^{\rm nd}$ Riemann sheet

$b(p_0) =$ 

$= b_0(p_0) + b_T(p_0)$ defined on the physical (upper) halfplane.

$$b_0^>(p_0) = \frac{1}{16\pi^2}\left[\ln\frac{m_\pi^2(T)}{M_0^2} - \sqrt{1-\frac{4m_\pi^2(T)}{p_0^2}}\left(\mathrm{arctanh}\sqrt{1-\frac{4m_\pi^2(T)}{p_0^2}} + i\pi\right)\right]$$

$$b_T^>(p_0) = \frac{1}{4\pi^2}\mathcal{P}\int_{\frac{m_\pi(T)}{T}}^{\infty} dx\,\frac{\sqrt{x^2-m_\pi^2(T)/T^2}}{p_0^2/4T^2-x^2}\frac{1}{\exp(x)-1} - \frac{i}{8\pi}\sqrt{1-\frac{4m_\pi^2(T)}{p_0^2}}\,n(\tfrac{p_0}{2})\Theta\left(p_0-2m_\pi(T)\right)$$

$b_T(p_0)$ has discontinuity along the real axis for $p_0 > 2m_\pi(T)$

Impose the continuity of $b(p_0)$ for $p_0 > 2m_\pi(T)$ across the real axis.

for complex p_0:

$$b_0^<(p_0) = b_0^>(p_0)$$

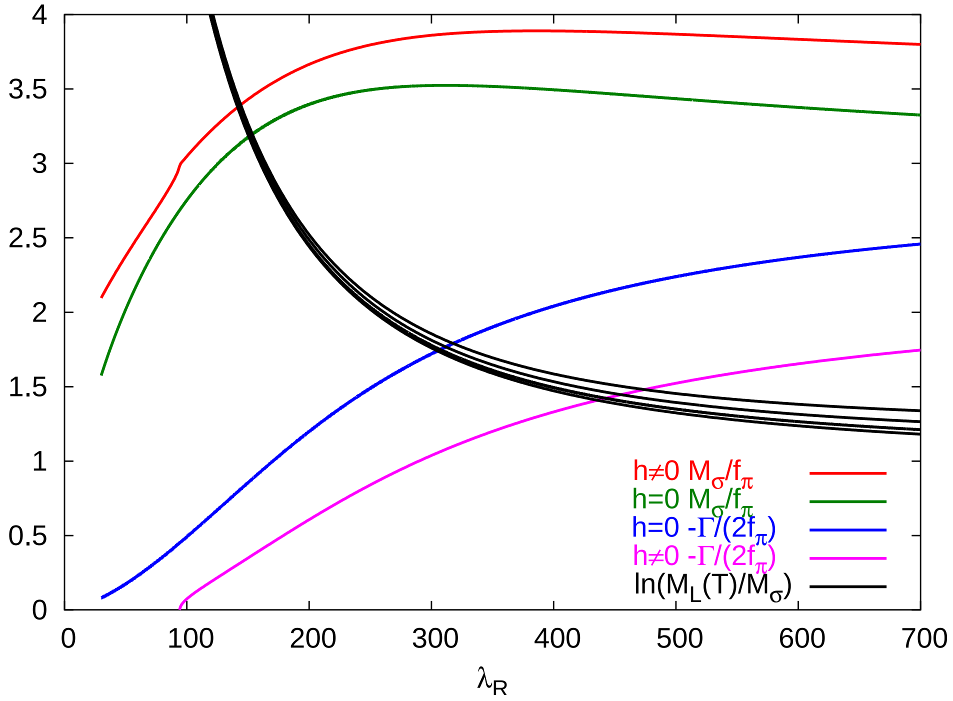
$$b_T^<(p_0) = b_T^>(p_0) - \frac{i}{4\pi}\frac{\sqrt{1-4m_\pi^2(T)/p_0^2}}{\exp(p_0/2T)-1}$$

Fixing $\boldsymbol{\lambda_R}$ with the $\boldsymbol{\sigma}$ pole at $\mathbf{T = 0}$

With the pole parametrisation $p_0 = 2m_{G0} + M_0 \exp(-i\varphi_0), \quad 0 < \varphi_0 < \pi$
one solves $G_\sigma^{-1}(p_0, \mathbf{p} = 0) = 0$ and determines $M_\sigma = \mathcal{R}ep_0,\, \Gamma = \mathcal{I}mp_0$

good value of λ_R: for which we get closest to the phenomenological values

constraint on λ_R: tachyonic pole on the imaginary axis of the physical sheet



$$m_\pi(0) = 0 \quad \lambda_R = 310$$
$$M_\sigma/\Gamma \sim 1$$
$$M_\sigma = 3.5 f_\pi$$

$$m_\pi(0) \neq 0 \quad \lambda_R = 400$$
$$M_\sigma/\Gamma = 1.4$$
$$M_\sigma = 3.95 f_\pi$$

A light and not broad enough σ is accessible in LO large N approximation.

Temperature driven pole trajectory

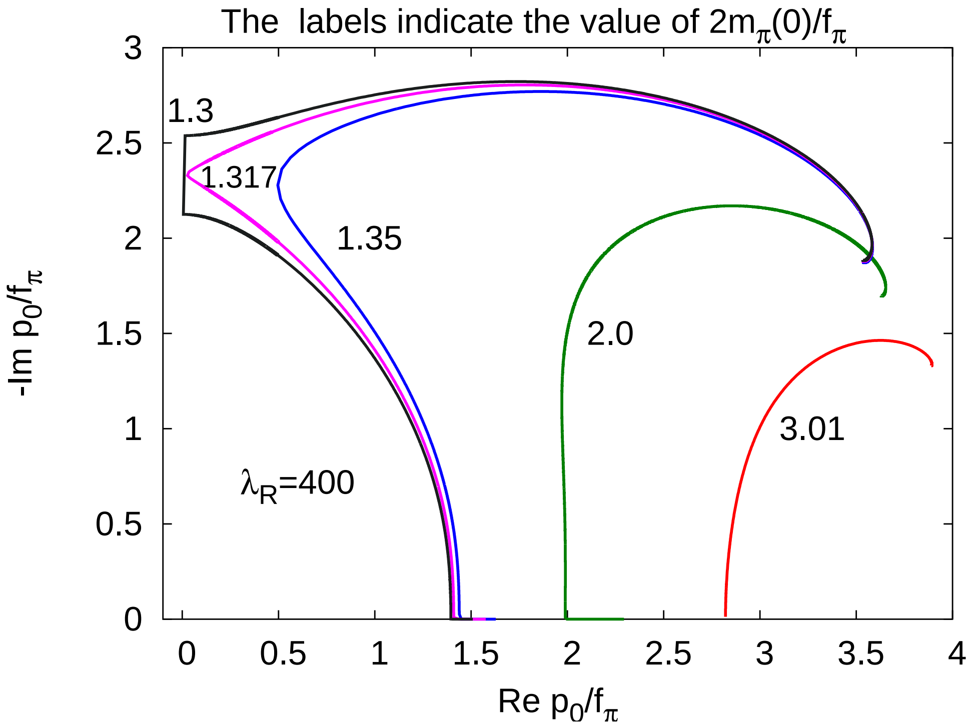
The σ-pole is obtained on the 2$^{\rm nd}$ Riemann sheet as the solution of:

$$\left(p_0^2 - m_\pi^2(T)\right)\left(1 - \frac{\lambda_R}{6}\left(b_0^<(p_0) + b_T^<(p_0)\right)\right) - \frac{\lambda_R}{3}\Phi^2(T) = 0.$$

Basic feature: the imaginary part of the pole will eventually decrease with increasing T and the pole approaches the threshold.

However, in the early stage the imaginary part is actually increasing.

The value of $m_\pi(0)$ tunes the trajectory of the pole.



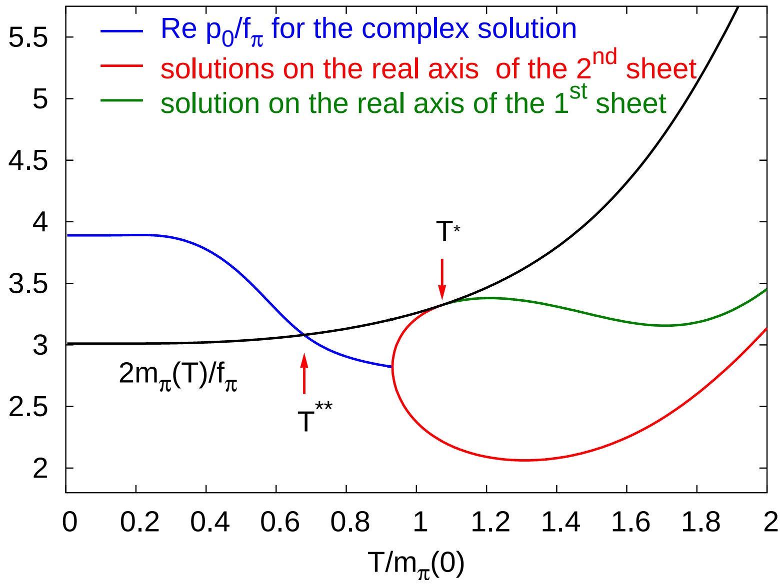
With decreasing $m_\pi(0)$ the pole approaches the imaginary axis.

Below $m_\pi(0) = 61$MeV it moves a while on the imaginary axis before switching over to the real axis.

In the chiral limit the pole goes to the origin along the imaginary axis.

$m_\pi(0) = 140$ MeV

Pole trajectory



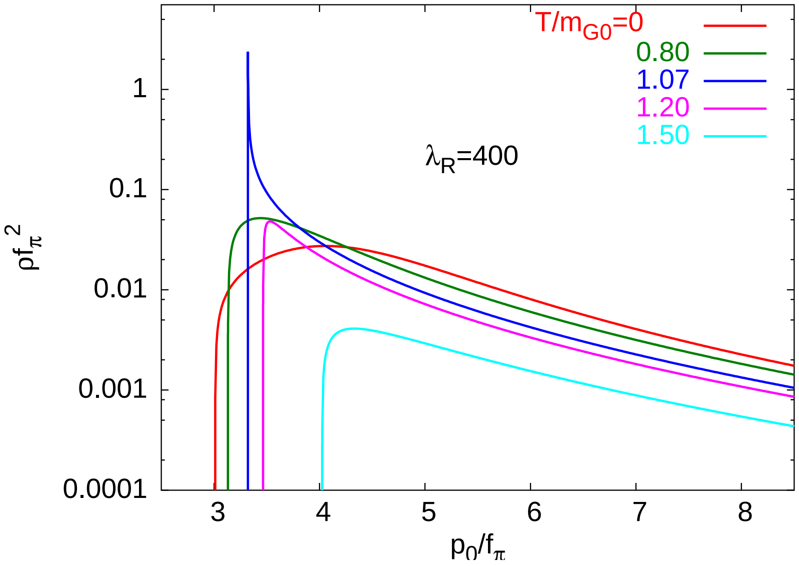
$2m_\pi(T)$ increases with T

$T^{**} \approx 0.69 m_\pi(0)$: real part of the 4th quadrant pole goes below the threshold.

It collides with its 1st quadrant mirror and splits up in two real poles.

$T^{*} \approx 1.07 m_\pi(0)$: One of the poles goes over the 1st Riemann sheet and describe a stable particle

Spectral function



threshold enhancement occurs for $T \in (T^{**}, T^*)$

In this interval the spectral function takes its maximum close to the threshold.

ρ_σ reflects the characteristics of the $T = 0$ σ pole only for $T < T^{**}$

Threshold enhancement

Scalar-isoscalar spectral function:

$$\rho_\sigma(p_0,\mathbf{p}=0,T)=$$

$$\frac{\lambda_R^2\Phi^2(T)\mathrm{Im}b_R^>(p_0)/18\pi}{\left[\left(p_0^2-m_\pi^2(T)\right)\left(1-\frac{\lambda_R}{6}\mathrm{Re}b_R^>(p_0)\right)-\frac{\lambda_R}{3}\Phi^2(T)\right]^2+\left[\left(p_0^2-m_\pi^2(T)\right)\frac{\lambda_R}{6}\mathrm{Im}b_R^>(p_0)\right]^2}$$

The second term in the denominator vanishes more slowly as $p_0\rightarrow 2m_\pi(T)$

near threshold behaviour of the two terms in the denominator:

$$\left(p_0^2-m_\pi^2(T)\right)\left(1-\tfrac{\lambda_R}{6}\mathrm{Re}b_R^>(p_0)\right)-\tfrac{\lambda_R}{3}\Phi^2(T)\approx p_0^2-4m_\pi^2(T)$$

$$\left(p_0^2-m_\pi^2(T)\right)\tfrac{\lambda_R}{6}\mathrm{Im}b_R^>(p_0)\approx\left[p_0^2-4m_\pi^2(T)\right]^{1/2}$$

Near the threshold:

$$\rho_\sigma(p_0,\mathbf{0},T^*)\sim\frac{1}{\sqrt{1-\frac{4m_\pi^2(T^*)}{p_0^2}}}$$

Universality near the pole-threshold coincidence

Assumptions: π stable at high temperature ⟶ σ is stable also

$G_\sigma(p_0)$ analytical around T^* and near the threshold

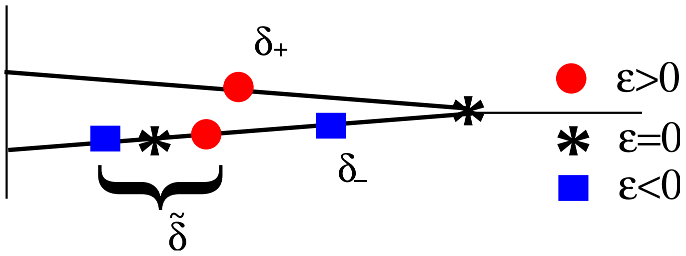
$$\begin{aligned} -G_\sigma^{-1}(p_0) &\approx a_1\epsilon + \tilde{a}_2\epsilon^2 + b_1\delta + b_2\delta^2 + c\delta\epsilon & \epsilon &:= (T-T^*)/T^* \\ &\approx \frac{1}{1+c\epsilon/b_1}\left[a_1\epsilon + a_2\epsilon^2 + b_1\delta + b_2\delta^2\right] & \delta &:= \sqrt{1-\frac{p_0^2}{4m_\pi^2}} \end{aligned}$$

performing analytical continuation from below to above the threshold

for $p_0 < 2m_\pi$ G_σ^{-1} is real

for $p_0 > 2m_\pi$ δ is imaginary ⟶ the spectral function is determined

for $a_1 < 0, b_{1,2} > 0$ only, we have δ_+ for $\epsilon > 0$ $\delta_-, \tilde{\delta}$ for $\epsilon < 0$



Coefficients determined from the large-N formulas

Spectral function

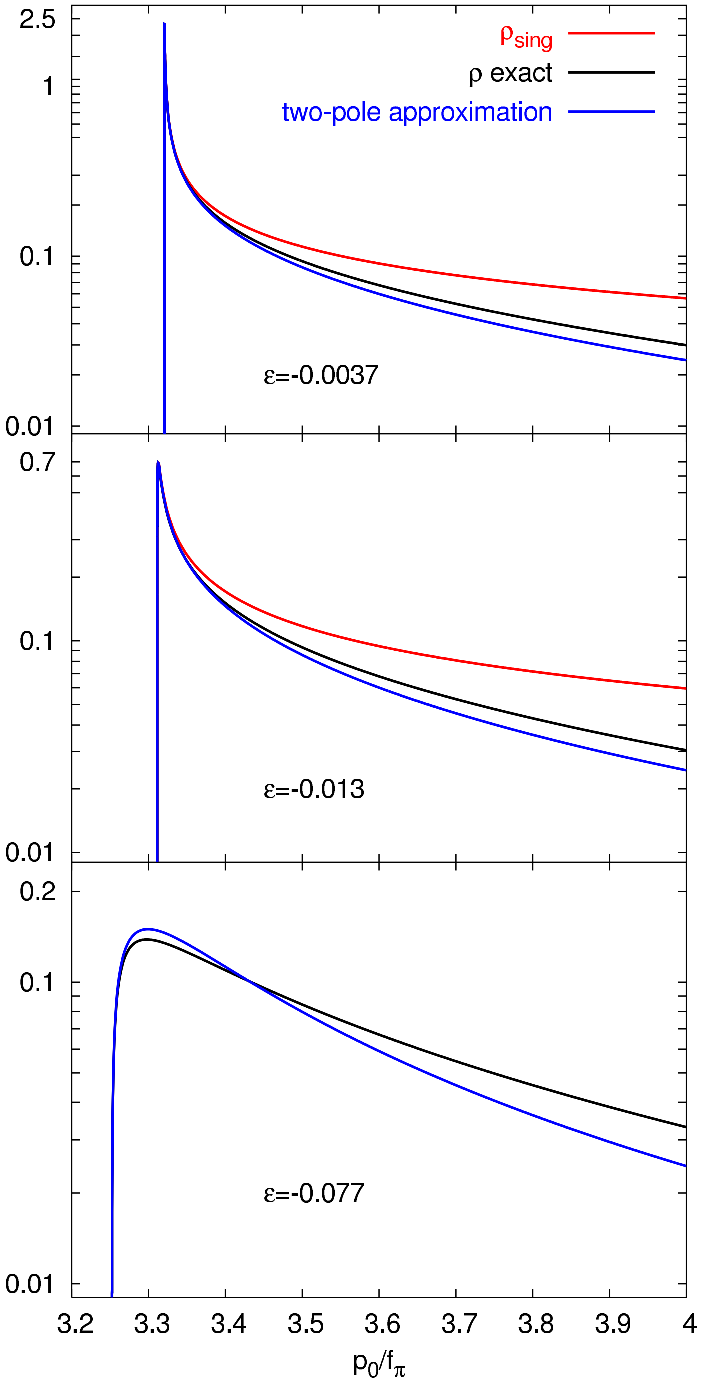
two poles approximation

$$\rho(p_0,\epsilon) = -\frac{\left[1+\frac{c}{b_1}\epsilon\right](b_2\pi)^{-1}\sqrt{\frac{p_0^2}{m_\pi^2}-1}(\tilde{\delta}+\delta_\pm)}{(\frac{p_0^2}{m_\pi^2}-1-\delta_\pm\tilde{\delta})^2+(\frac{p_0^2}{m_\pi^2}-1)(\tilde{\delta}+\delta_\pm)^2}$$

one pole approximation

$$\rho_{\rm sing}(p_0,\epsilon) = \frac{(1+c\epsilon/b_1)}{b_2(\delta_\pm-\tilde{\delta})\pi}\times\frac{|\delta|}{|\delta|^2+\delta_\pm^2}.$$

A. Patkós, Zs. Sz., P. Szépfalusy,
Phys.Rev. **D** 68 (2003) 047701

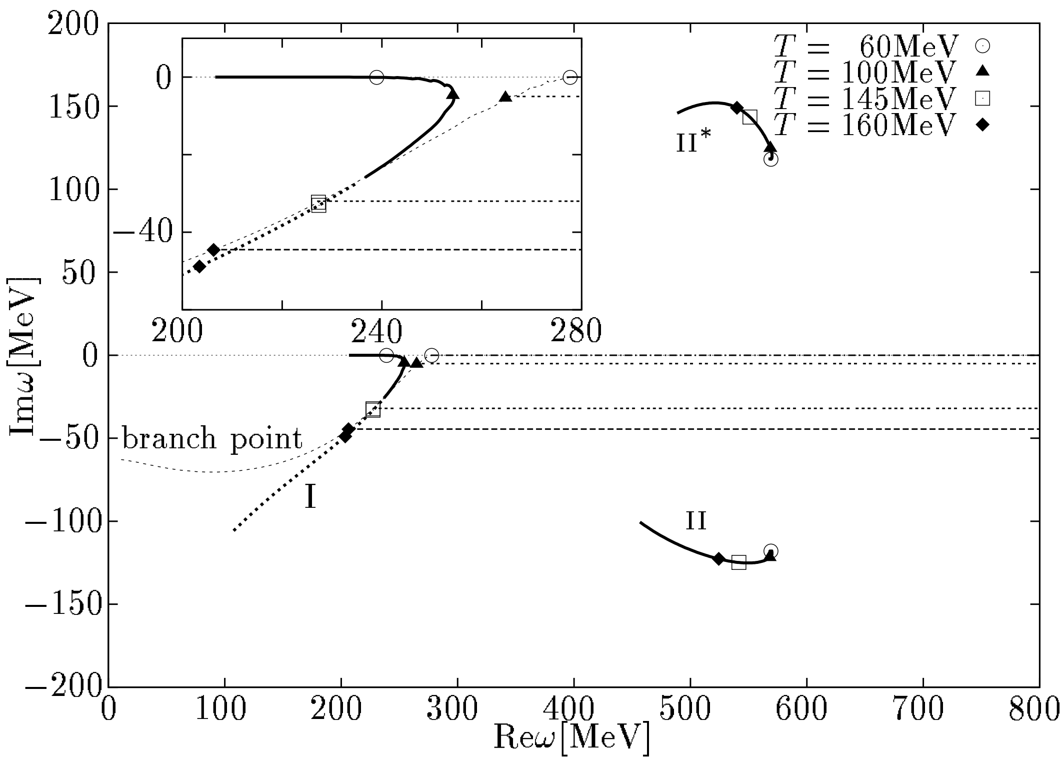


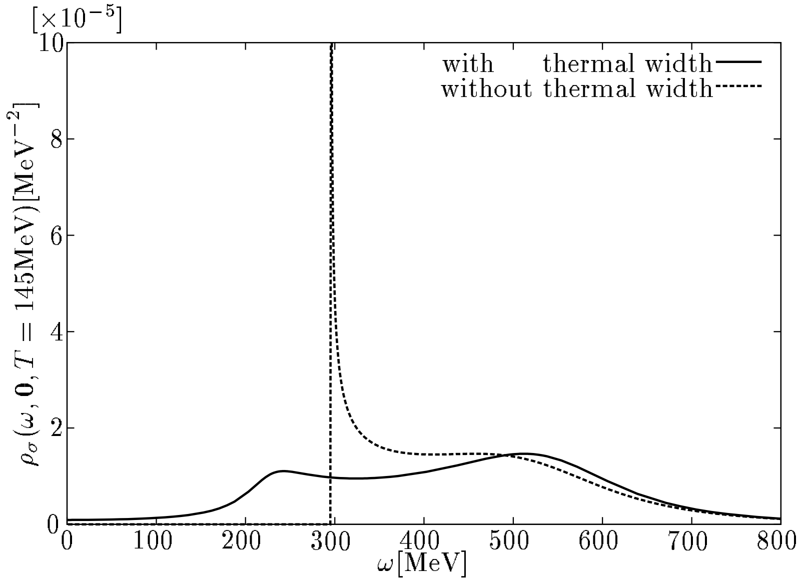
Effect of pion width on the sigma spectral function

Y. Hidaka, O. Morimatsu, T. Nishikawa, M. Ohtani, hep-ph/0304204

$\varGamma_\pi \approx 50$ MeV thermal width assumed from $\pi + \pi^{\mathrm{thermal}} \to \sigma$ scattering

$$D_\pi^{-1} = p^2 - m_{0\pi}^2 - \Pi_\pi(p^2, \mathbf{p}, T)\Big|_{p^2=(m_\pi^{\mathrm{pole}})^2} = 0 \qquad m_\pi^{\mathrm{pole}} = m_\pi^*(T) - i\frac{\varGamma_\pi(T)}{2}$$





The linear sigma model with chiral fermions

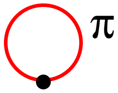
$$L = L(\sigma,\pi) + \bar{\psi}(x)\left[i\partial^\mu\gamma_\mu - m_F - \frac{g}{\sqrt{N}}\left(\sigma(x) + i\sqrt{2N_f}\gamma_5 T^a\pi^a(x)\right)\right]\psi(x)$$

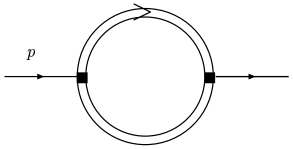
chemical potential: $i\partial_t\Psi \to (i\partial_t + \mu)\Psi$

it is required that $m_F = g\Phi$ stays finite as $N_f = N^{1/2} \to \infty$

fermion contribution is $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$

Method: fermions taken into account perturbatively (contribution with lowest power of g only) both in the EoS and in the pion propagator

pion pole: $m^2 + \frac{\lambda}{6}\Phi^2 +$ 

$+$ 

$\Big|_{p^2=M^2} = M^2$

EoS: $m^2 + \frac{\lambda}{6}\Phi^2 + \frac{\lambda}{6N}\Phi\langle\pi^a\pi^a\rangle + \frac{g}{N}\langle\bar{\psi}\psi\rangle = h$

on the lhs pion propagator is parameterised with M^2

The phase diagram in the chiral limit, $h = 0$

Preliminary result   A. P, Zs. Sz., P. Sz, A. Jakovác

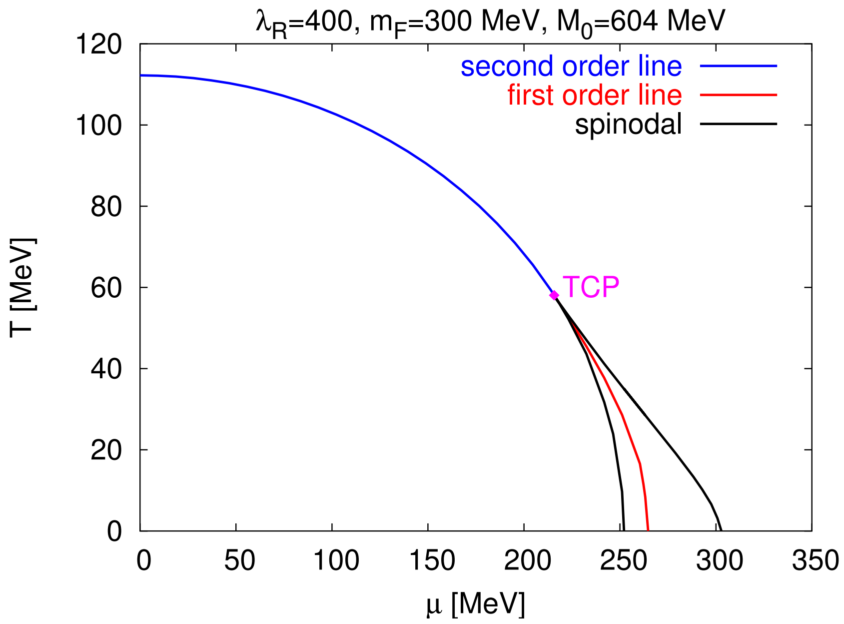
Gap equation: $M^2[1 - g^2 N_c B(M, m_N)] = 0 \longrightarrow M = 0$ Goldstone theorem

EoS: $$\Phi\left[m_R^2 + \frac{\lambda_R}{6}\Phi^2 + \frac{\lambda_R}{6} I^{\pi}_{tad}(M=0) - 4N_c\frac{g^2}{\sqrt{N}} I^{\Psi}_{tad}(m_N)\right] = 0$$

analytical determination of the 2nd order line and of the tricritical point (TCP):

$$\left.\begin{aligned} m_R^2 + \frac{\lambda_R}{72}T_c^2 - \frac{g^2 T_c^2}{2\pi^2} N_c \left(Li_2(-e^{\mu/T_c}) + Li_2(-e^{-\mu/T_c})\right) &= 0 \\ \frac{\lambda_R}{6} + \frac{g^4 N_c}{4\pi^2}\left[\frac{\partial}{\partial n}\left(Li_n(-e^{\mu/T_c}) + Li_n(-e^{-\mu/T_c})\right)\Big|_{n=0} - \ln\frac{cT_c}{M_0}\right] &= 0 \end{aligned}\right\} \Rightarrow \{\mu, T\}_{TCP}$$

$$\ln c/2 = 1 - \gamma$$



work in progress: TCP $\longrightarrow$ CEP for $h \neq 0$

- its motion with h?
- realistic σ-pole position in presence of fermion vacuum fluctuation?
- why is T_c so small?
  (see also Scavenius et al,
  Phys. Rev. C **64** (2001) 045202)