Paul Romatschke

Collective Modes of an Anisotropic Quark-Gluon Plasma

PR & M. Strickland, PRD68, 036004; hep-ph/0309093
Inst.f.Theoretical Physics, TU Vienna

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Gluon polarization tensor

- Within semi-classical transport theory, the color current induced by a soft gauge field is given by [1]

$$J^{\mu,a}_{\text{ind}}(X) = g \int \frac{d^3p}{(2\pi)^3} V^\mu [2N_c \delta N^a(p,X) + N_f(\delta n^a_+(p,X) - \delta n^a_-(p,X))] \,, \quad (1)$$

  to leading order in the coupling g, where $V^\mu = (1, \mathbf{k}/\omega)$ is the gauge field four-velocity.

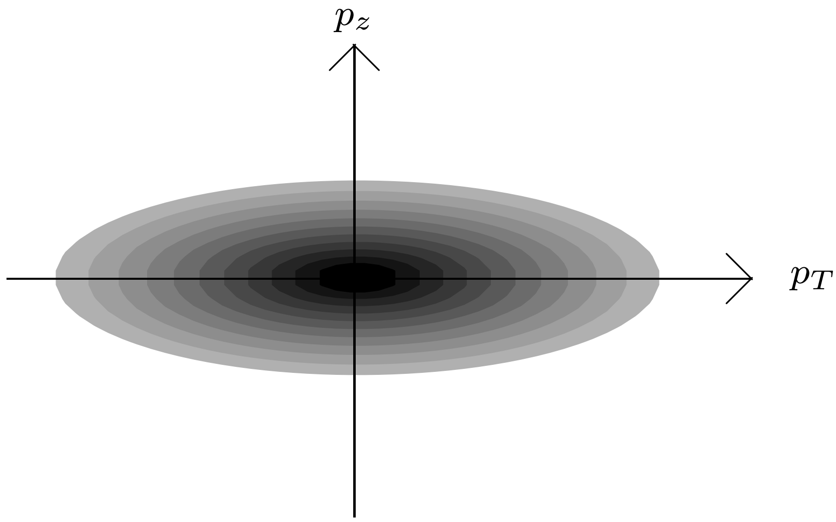
- The fluctuating parts of the gluon and quark/anti-quark densities $\delta N^a(p,X)$ and $\delta n^a_+(p,X), \delta n^a_-(p,X)$, respectively, satisfy the transport equations

$$\begin{aligned} [V \cdot D_X, \delta n_\pm(p,X)] &= \mp g V_\mu F^{\mu\nu}(p,X) \partial_\nu n_\pm(\mathbf{p}), \\ [V \cdot D_X, \delta N(p,X)] &= -g V_\mu F^{\mu\nu}(p,X) \partial_\nu N(\mathbf{p}), \quad (2) \end{aligned}$$

  where $D_X = \partial_X + igA(X)$ is the covariant derivative.

- At this point the tree-level densities $N(\mathbf{p})$ and $n_\pm(\mathbf{p})$ are completely arbitrary. However, to obtain explicit results, it is assumed [3] in the following that these initial density distributions are anisotropic in the sense that they can be obtained from the standard isotropic densities by rescaling of only one direction in momentum space, e.g.

$$N(\mathbf{p}) = N_{\text{iso}} \left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p} \cdot \hat{\mathbf{n}})^2} \right). \quad (3)$$



Contour plot of $N(\mathbf{p})$ with positive anisotropy parameter ($\xi > 0$). The anisotropy vector $\hat{\mathbf{n}}$ is taken to lie along the p_z-direction.

- Solving the transport equations and neglecting terms of sub-leading order in g one obtains the gluon polarization tensor

$$\Pi^{\mu\nu}(K)=g^2\int\frac{d^3p}{(2\pi)^3}V^\mu\partial^\beta_{(p)}f(\mathbf{p})\left(g_{\nu\beta}-\frac{V_\nu K_\beta}{K\cdot V+i\epsilon}\right) \quad (4)$$

via the relation $\Pi^{\mu\nu}(K)=\frac{\delta J^\mu_{\rm ind}(K)}{\delta A_\nu(K)}$ and $f(\mathbf{p})=2N_cN(\mathbf{p})+N_f(n_+(\mathbf{p})+n_-(\mathbf{p}))$.

- It can be shown that the tensor $\Pi^{\mu\nu}$ is both symmetric $\Pi^{\mu\nu}=\Pi^{\nu\mu}$ and transverse $K_\mu\Pi^{\mu\nu}=0$. Accordingly, not all components are independent and one can restrict the following considerations to the spatial part Π^{ij}.

- The spatial part of the propagator Δ^{ij} in the temporal axial gauge ($A_0=0$) can then be calculated to be

$$\Delta^{-1}(K))^{ij}=[(k^2-\omega^2)\delta^{ij}-k^ik^j+\Pi^{ij}(K)]. \quad (5)$$

The dispersion relations for the collective modes can be obtained by finding the poles of the propagator $\Delta^{ij}(K)$.

Tensor decomposition

- In an anisotropic system, there exists a preferred direction. Accordingly, to decompose Π^{ij}, one needs a basis for a symmetric 3-tensor that depends on the momentum k^i as well as the anisotropy 3-vector n^i. This basis can be constructed explicitely as

$$\begin{aligned} A^{ij} &= \delta^{ij} - k^i k^j / k^2, \\ B^{ij} &= k^i k^j / k^2, \\ C^{ij} &= \tilde{n}^i \tilde{n}^j / \tilde{n}^2, \\ D^{ij} &= k^i \tilde{n}^j + k^j \tilde{n}^i, \end{aligned} \tag{6}$$

  where $\tilde{n}^i = A^{ij} n^j$ which obeys $\tilde{n} \cdot k = 0$.

- Using this tensor basis, Π^{ij} is decomposed into four structure functions

$$\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij}, \tag{7}$$

  which are determined by considering the contractions

$$\begin{aligned} k^i \Pi^{ij} k^j &= k^2 \beta \ , \\ \tilde{n}^i \Pi^{ij} k^j &= \tilde{n}^2 k^2 \delta \ , \\ \tilde{n}^i \Pi^{ij} \tilde{n}^j &= \tilde{n}^2 (\alpha + \gamma) \ , \\ \mathrm{Tr}\, \Pi^{ij} &= 2\alpha + \beta + \gamma \ . \end{aligned} \tag{8}$$

Static limit

- Using the tensor decomposition, the propagator takes the form

$$\boldsymbol{\Delta}(K) = \Delta_A\,[\mathbf{A}-\mathbf{C}]+\Delta_G\,[(k^2-\omega^2+\alpha+\gamma)\mathbf{B}+(\beta-\omega^2)\mathbf{C}-\delta\mathbf{D}]\ , \tag{9}$$

where in the limit $\omega \to 0$ Δ_A and Δ_G become

$$\begin{aligned} \Delta_A^{-1} &= k^2 + m_\alpha^2, \\ \Delta_G^{-1} &= -\frac{\omega^2}{k^2}(k^2+m_+^2)(k^2+m_-^2) \end{aligned} \tag{10}$$

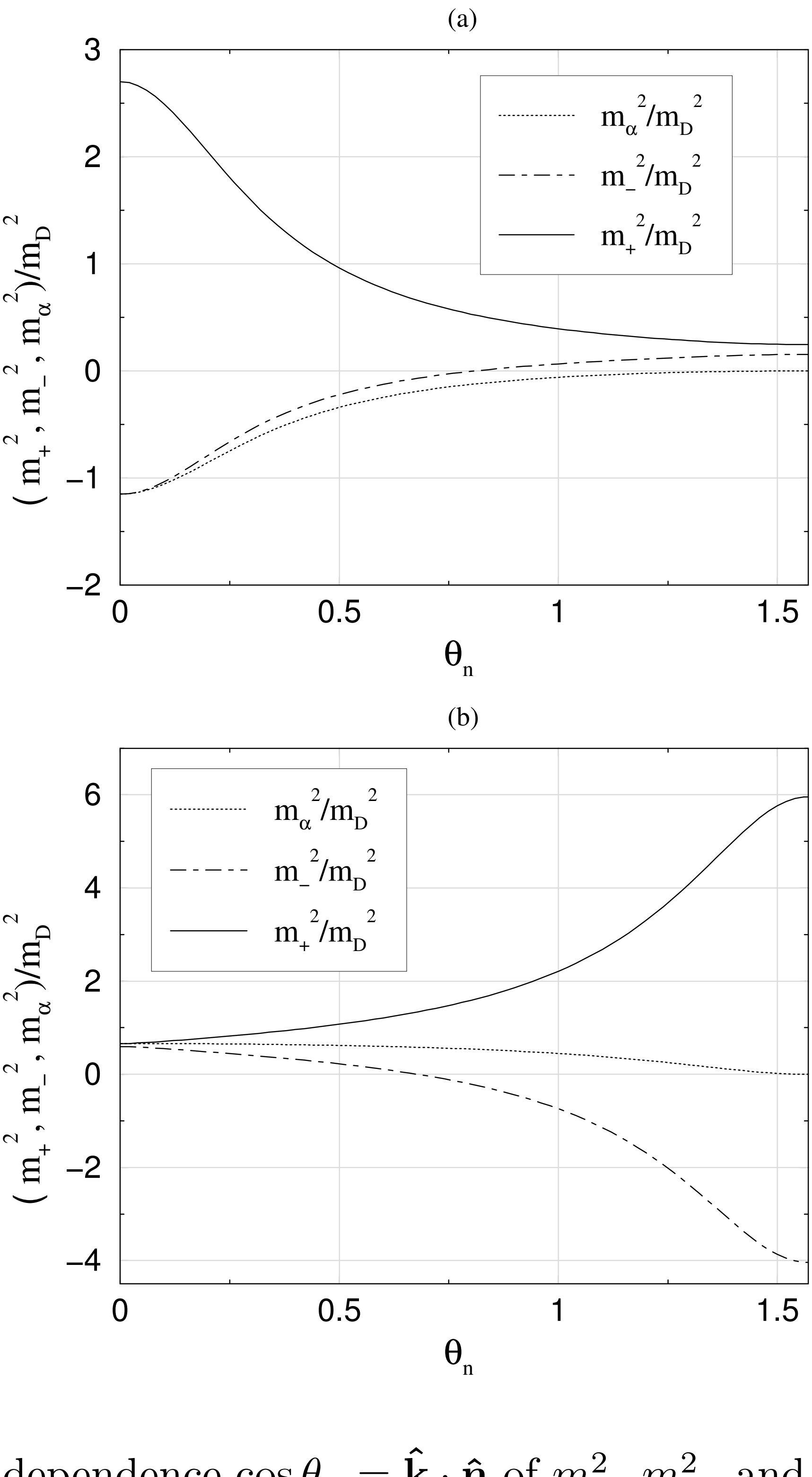
and where

$$2m_\pm^2 = M^2 \pm \sqrt{M^4 - 4(m_\beta^2(m_\alpha^2+m_\gamma^2) - m_\delta^4)} \tag{11}$$

with $M^2 = m_\alpha^2 + m_\beta^2 + m_\gamma^2$ and the mass scales are defined as

$$\begin{aligned} m_\alpha^2 = \lim_{\omega\to 0}\alpha, \quad m_\beta^2 = \lim_{\omega\to 0} -\tfrac{k^2}{\omega^2}\beta\ , \\ m_\gamma^2 = \lim_{\omega\to 0}\gamma, \quad m_\delta^2 = \lim_{\omega\to 0}\tfrac{\tilde{n}k^2}{\omega}\mathrm{Im}\,\delta\ . \end{aligned} \tag{12}$$

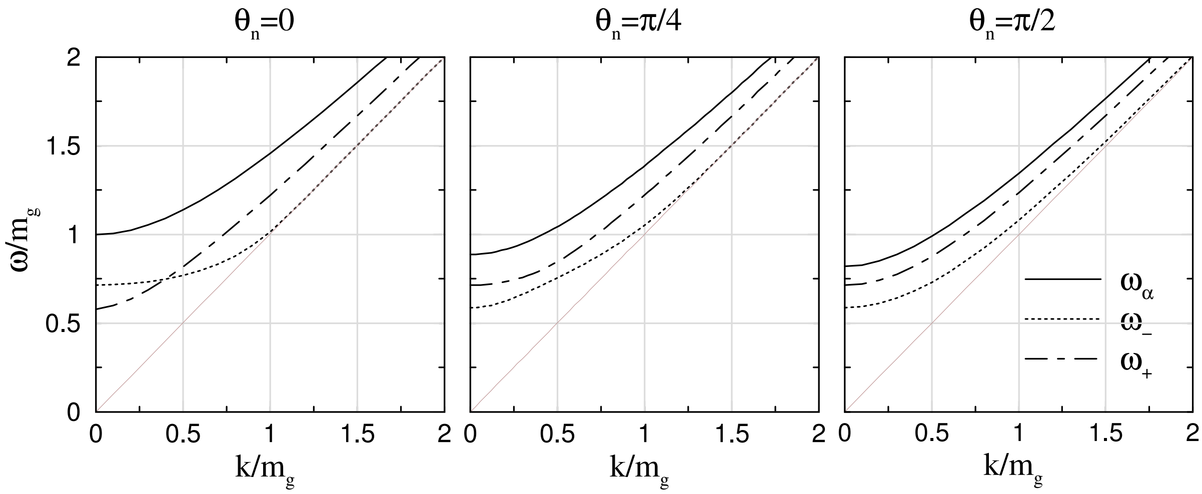
- For some angles $\cos\theta_n = \hat{\mathbf{k}}\cdot\hat{\mathbf{n}}$, m_-^2, m_α^2 turn out to be negative, signaling the presence of unstable modes, as has been noted in Refs. [2, 3, 4]. As has been pointed out [2], the plasma instabilities associated with these modes are known as Weibel [5] or pinching instabilities.



Angular dependence $\cos\theta_n = \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$ of m_α^2, m_+^2, and m_-^2 at fixed (a) $\xi = 10$ and (b) $\xi = -0.9$; m_D is the Debye mass.

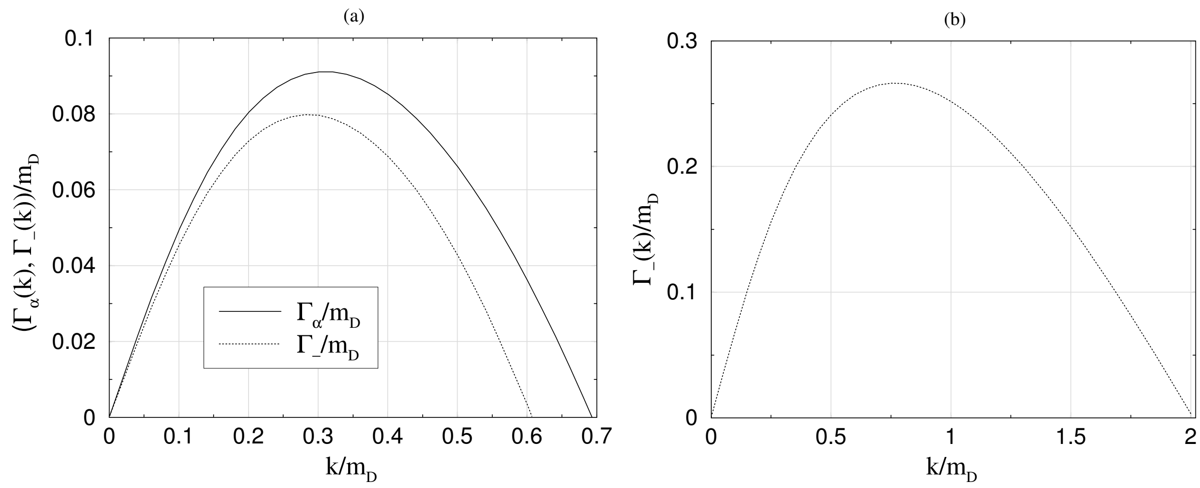
Collective modes

In the non-static case Δ_G^{-1} is also factorizable, allowing a determination of the dispersion relations for all the collective modes in the system.
Stable modes: For real-valued $\omega > k$ one finds two stable modes coming from Δ_G and one coming from Δ_A.



Angular dependence of the dispersion relations of the stable modes $\omega_\alpha, \omega_+, \omega_-$ for $m_g = \frac{m_D}{\sqrt{3}}$, $\xi = 10$ and $\theta_n = \{0, \frac{\pi}{4}, \frac{\pi}{2}\}$.

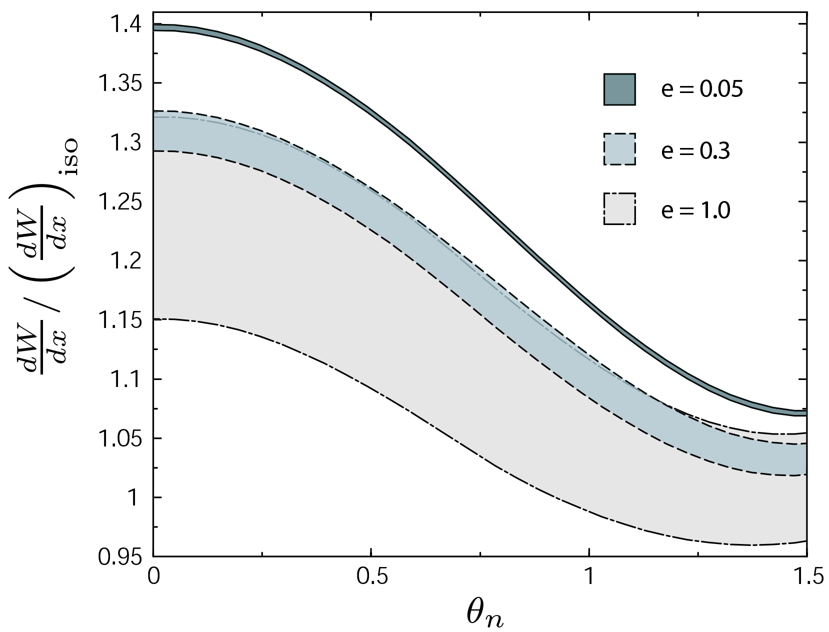
Unstable modes: For nonzero anisotropy parameter ξ the propagators also have poles for imaginary ω. The growth rate of these can be determined by taking $\omega \to -i\Gamma$ and solving for $\Gamma(k)$. Depending on the angle θ_n one finds that for positive ξ there can be one or two unstable modes while for negative ξ there is at most one unstable mode.



Growth rates Γ_α, Γ_- as a function of k with (a) $\xi = 10$ and $\theta_n = \frac{\pi}{8}$ and (b) $\xi = -0.9$ and $\theta_n = \frac{\pi}{2}$.

Collisional energy loss

Following a technique by Braaten & Thoma the collisional energy loss of a heavy fermion for QED with an anisotropic electron momentum-space distribution function has recently been calculated (PR & MS, hep-ph/0309093). The result indicates a pronounced directional dependence of the energy loss if the momentum-space anisotropy is strong.



Energy loss scaled by isotropic result for a heavy fermion in QED where $\cos\theta_n = \hat{\mathbf{v}} \cdot \hat{\mathbf{n}}$ is the angle of the fermion propagating with velocity $v = 0.7$ with respect to the anisotropy vector ($\xi = 10$).

Conclusions

- The collective modes of a quark-gluon plasma for a special class of anisotropic momentum-space distribution functions were analyzed.
- One finds three stable modes and up to two unstable modes, which may play an important role in the dynamical evolution of the quark-gluon plasma.
- As a first application, the collisional energy loss in an anisotropic QED plasma was calculated [3], indicating a pronounced directional dependence of the energy loss for large ξ. The extension for QCD is work in progress.

References

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