Renormalization in 2PI resummation schemes

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Introduction

- Asymptotic freedom $\Rightarrow$ description of the thermalized quark-gluon plasma in perturbation theory?
- Really possible very far from the transition temperature. When extrapolating close to the transition regime, the perturbative series looses its predictive power.
- On the other hand, quasiparticles models reproduce lattice simulations up to a few times $T_c \Rightarrow$ effect of interactions is essentially a dressing of degrees of freedom.
- Phenomenological models need to be recovered from first principles (QCD Lagrangian). Quasiparticle picture $\Rightarrow$ main role played by the full propagator.

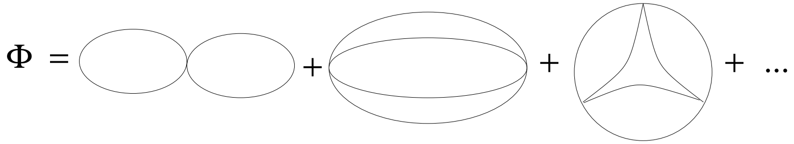
Outline

- Introduction
- Φ-derivable approximations
- Analysis of UV singularities
  - Wave function renormalization
  - Coupling renormalization
  - Mass renormalization
- How to implement renormalization?
- Thermal effects
- Massless case
- Conclusions

Φ-derivable approximations

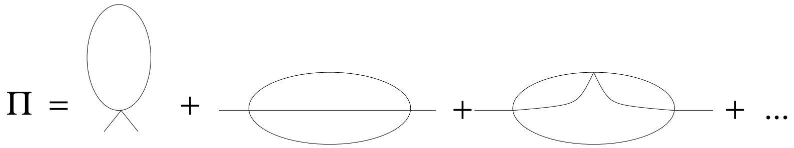
- Thermodynamical quantities expressed in terms of D :

$$\beta\Omega[D] = \frac{1}{2}\mathrm{Tr}\ \ln D^{-1} - \frac{1}{2}\mathrm{Tr}\ \Pi D + \Phi[D]$$



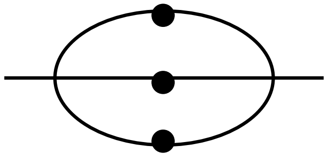
- D itself defined via a stationnarity condition

$$\frac{\delta\Omega}{\delta D} = 0 \leftrightarrow \Pi = 2\frac{\delta\Phi}{\delta D}$$

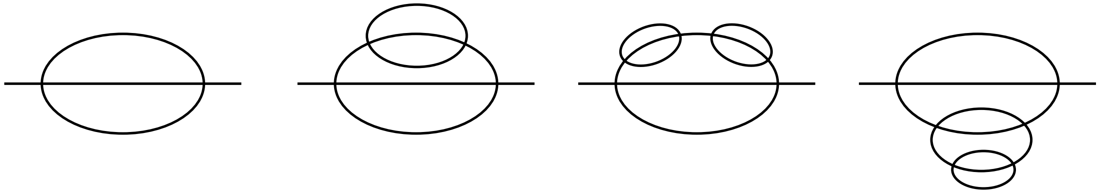


- Φ-derivable approximations preserve stationnarity

Renormalization in resummed schemes

$\Pi =$  $+ K^2\delta Z+\delta m^2$

- Infinite set of Feynman diagrams resummed



$\Rightarrow$ Infinite set of UV singularities
Is it possible to absorb them into $\delta\lambda$, δm^2 and δZ?

- Renormalization cannot be naive: $\delta\lambda$, δm^2 and δZ cannot depend on Π because T will enter the game via Π

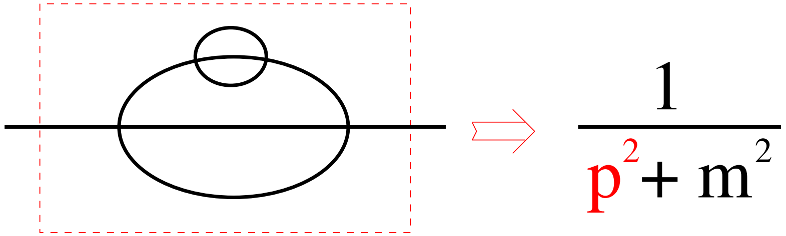
Toolkit

- $\delta\lambda$, δm^2 and δZ determined by a consistency condition: suppose Π finite and tune the c.t. in such a way that the right hand side of the gap eq. does not generate UV singularities.
- Iterations $\Rightarrow$ perturbative expansion of the solution $\Rightarrow$ Standard BPH analysis on perturbative diagrams and Weinberg's theorem

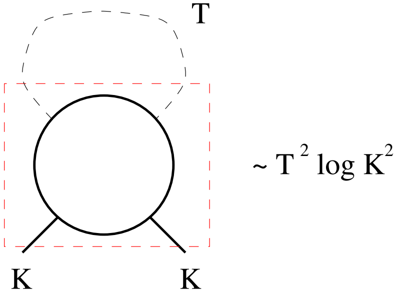
Asymptotic behaviour of Π

$\Pi_2(K) = K^2 F(\ln K^2/\mu^2)$, F smooth function

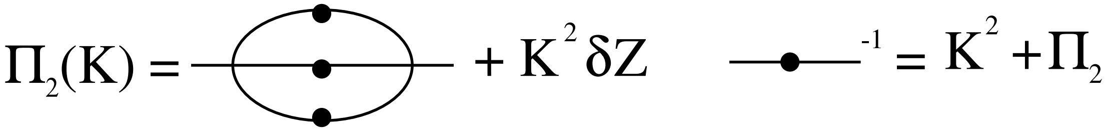
- depends only on the renormalization scale μ



- does not depend on in-medium effects (T)



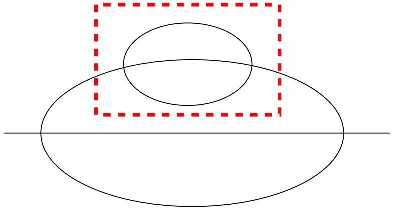
- simplified gap equation



Wave function renormalization

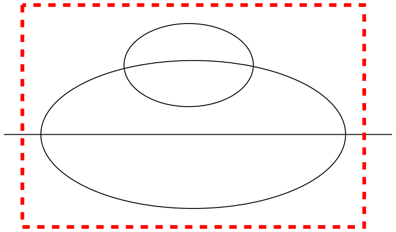
Iterate the equation and draw all the 2-point boxes

- boxes drawn around insertions



already renormalized by hypothesis

- 2 PI ⇒ only one remaining 2-point box: the diagram itself



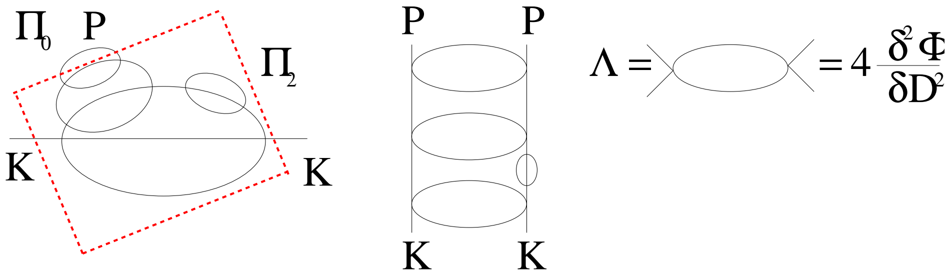
⇒ global divergence ⇒ local divergence ⇒ $K^2\delta Z$

- δZ remains the same if we add m or T

Coupling singularities

Iterate the equation and draw all the 4-point boxes

- the new coupling singularities are essentially obtained by adding a Λ to a Bethe Salpether tower



- Simplified equation for coupling singularities

$$\Gamma(K,P) = \Lambda(K,P) - \frac{1}{2}\int_Q \Lambda(K,Q) D_2^2(Q) \Gamma(Q,P)$$

- In terms of insertions, the one with a line outside the box contributes through its subleading part Π_0

Coupling renormalization

- 2 PI $\Rightarrow \Lambda(K,P) \sim AF(\ln P)$ for large P and fixed K
  $\Rightarrow \Lambda(K,P) - \Lambda(0,P) \sim K/P$
- Locality

$$\begin{aligned} \Gamma(K,P) - \Gamma(0,P) &= \Lambda(K,P) - \Lambda(0,P) \\ &- \frac{1}{2}\int_Q \left\{\Lambda(K,Q) - \Lambda(0,Q)\right\} D_2^2(Q)\Gamma(Q,P \end{aligned}$$

$\Rightarrow \Gamma$ renormalized via a local counterterm $\delta\lambda$

- The value of $\delta\lambda$ depends on D only via D_2

Mass renormalization

- $\delta\lambda$, δZ and $\frac{\delta m^2}{m^2} = C$ can be taken m^2-independent
- by deriving the gap equation with repect to m^2

$$\frac{\partial \Pi}{\partial m^2}(K) = \frac{1}{2}\int_P \Lambda(K,P)\left\{1 + \frac{\partial \Pi}{\partial m^2}(P)\right\} D^2(P) + C$$

- $\Lambda(K,P) - \Lambda(0,P) \sim K/P \to$ C is local
- by trading Λ for Γ via the BS equation

$$\frac{\partial \Pi}{\partial m^2}(K) = \frac{1}{2}\int_P \Gamma(K,P) D^2(P) + C\left\{1 - \frac{1}{2}\int_P \Gamma(K,P) D^2(P)\right\}$$

- C depends on D, only via D_2

Implementing renormalization

- Separate wave function from coupling and mass singularities

$$
\begin{aligned}
D(P) &= D_2(P) + \delta D(P) \\
\delta D(P) &= -\Pi_0(P) D_2^2(P) + D_r(P)
\end{aligned}
$$

$$\Pi(K) = \tilde{\Pi}_2(K) + \frac{1}{2}\int_P \Lambda(K,P)\delta D(p) + \Pi_r(K)$$

$D_2 \sim 1/P^2$, $\Pi_0(P) D_2^2 \sim 1/P^4$, $D_r \sim 1/P^6$, $\Pi_r \sim 1/K^6$

- $\frac{1}{2}\int_P \Lambda(K,P)\Pi_0(P)D_2^2(P)$ contains all the coupling singularities generated by the BS equation

Implementing renormalization II

- Coupling renormalization $\Leftrightarrow$ trade bare (Λ) for finite (Γ, Π_0-independent) quantities

$$\begin{aligned}
\Pi(K) &= \tilde{\Pi}_2(K) - \tilde{\Pi}_2(0) + \frac{1}{2}\int_P \{\Lambda(K,P) - \Lambda(0,P)\}\,\delta D(P) \\
&+ \frac{1}{2}\int_P \Gamma(0,P)\,\{D_r(P) - \Pi_r(P)D_2^2(P)\} \\
&+ \frac{1}{2}\int_P \Gamma(0,P)\left\{\Pi_2(P) - \tilde{\Pi}_2(P)\right\} D_2^2(P) + \tilde{\Pi}_2(0)
\end{aligned}$$

- Singularities isolated in the third line, Π_0-independent

$$\frac{m^2}{2}\int_P \Gamma(0,P)D_2^2(P) + \delta m^2 \left\{1 - \frac{1}{2}\int_P \Gamma(0,P)D_2^2(P)\right\}$$

Thermal effects

- Separate wave function from coupling and mass singularities

$$\begin{aligned} D(P) &= D_2(P) + \delta D(P) \\ \delta D(P) &= -\Pi_0(P) D_2^2(P) + D_r'(P) \end{aligned}$$

$$\Pi(K) = \tilde{\Pi}_2(K) + \frac{1}{2}\int_P \Lambda(K,P)\delta D(p) + \Pi_r'(K)$$

$D_r'(P) = D_r(P) + \rho(P)\epsilon(p_0)n(|p_0|)$ and $\Pi_r'(P) \sim 1/K^2$

- In imaginary time, no such a direct writing. Only obtained after performing the Matsubara sums and disentangling thermal and vacuum contributions.

Thermal effects II

- Coupling renormalization $\Leftrightarrow$ trade bare (Λ) for finite (Γ, T-independent) quantities

$$\begin{aligned}\Pi(K) &= \tilde{\Pi}_2(K) - \tilde{\Pi}_2(0) + \frac{1}{2}\int_P \{\Lambda(K,P) - \Lambda(0,P)\}\,\delta D(P) \\ &+ \frac{1}{2}\int_P \Gamma(0,P)\,\{D_r'(P) - \Pi_r(P) D_2^2(P)\} \\ &+ \frac{1}{2}\int_P \Gamma(0,P)\left\{\Pi_2(P) - \tilde{\Pi}_2(P)\right\} D_2^2(P) + \tilde{\Pi}_2(0)\end{aligned}$$

- Singularities isolated in the third line, T-independent

$$\frac{m^2}{2}\int_P \Gamma(K,P) D_2^2(P) + \delta m^2\left\{1 - \frac{1}{2}\int_P \Gamma(K,P) D_2^2(P)\right\}$$

Massless case

How to implement renormalization in the massless case?

- expansion around $D_2 \leftrightarrow$ expansion around vacuum

$$D = D_2 - \Pi_0 D_2^2 + D_r$$

in contradiction with the spirit of 2 PI resummation

- expansion spoiled by IR divergent quantities ($m \to 0$)

$$\Gamma \sim \ln m^2 \qquad \Pi_r \sim \frac{1}{m^2}$$

- Without modifying the analysis of singularities

$$D = D_2 - \Pi_0 D^2 + D_r'$$

Massless case II

$$\begin{aligned}\Pi(K) &= \Pi_2(K) - \frac{1}{2}\int_P \{\Lambda(K,P) - \Lambda(0,P)\}\,\delta D(P) + \Pi'_r(K) \\ &- \frac{1}{2}\int_P \Gamma(0,P) D'_r(P) - \frac{1}{2}\int_Q \Gamma(0,Q) D^2(Q) \Pi'_r(Q)\end{aligned}$$

IR safe.

Conclusions

- We showed how to renormalize Φ-derivable approximations in the imaginary-time formalism.
- We treated vaccum and thermal fluctuations in a same footing: starting from the vacuum sector, the correct renormalization of coupling singularities leads to no new UV difficulty when turning on temperature.
- We presented a renormalization program applicable for massless theories (relevant for hot field theories): we managed to keep thermal regulators in the propagators without introducing thermal counterterms.