Ladder qcd with finite isospin chemical potential

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- Qcd ground state and pion condensation
- Ladder qcd model
- Analysis of the phase diagram
- Conclusions

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Qcd ground state

Vacuum features

- In the chiral limit ($m_q = 0$) chiral transformations are an exact symmetry of the lagrangian of the theory

- $\langle \bar{\Psi}\Psi \rangle \neq 0$ (dynamical effect+explicit) not invariant

In the SU(2) (SU(3)) flavor sector dynamical effect is bigger than explicit$\rightarrow$ dynamical symmetry breaking

$\Downarrow$

pions (kaons)$\leftrightarrow$ Goldstone bosons (approximate)

Extension at finite temperature and densities (equilibrium formalism)

- Analysis of the ground state at $T, \mu \neq 0$:
  we need to test whether the free energy of the system is lowered when a field acquire a non zero expectation value ($\rightarrow$ variational approach)

$\Downarrow$

Qcd phase diagram

- Methods
  Lattice (problems at finite densities)
  Effective models
  Matrix models

We define

$$\mu_B = (\mu_u + \mu_d)/2 \qquad \mu_I = (\mu_u - \mu_d)/2$$

For high μ_B we expect formation of di-quark fields $\langle qq \rangle \neq 0$ (color superconductivity, CFL)

Here we are interested on the case when $\mu_I \neq 0$

- Effects on the phase transition (1 or 2 transitions)
- μ_I can be introduced on the lattice ($\mu_I \neq 0 \leftrightarrow \mu_B \neq 0$)
- Compared to the case $\mu_I = 0$ we expect $\rho = \frac{1}{2}\langle \bar{u}\gamma_5 d - \bar{d}\gamma_5 u \rangle \neq 0$ if μ_I is higher than half of the pion mass

$$\pi^+ \sim u\bar{d} \;\Rightarrow\; \mu_\pi \leftrightarrow 2\mu_I$$

The introduction of the chemical potential for a meson produces a negative mass term

- Condensation when $\mu_\pi = 2\mu_I \geq m_\pi$
- Order parameter $\rho = \frac{1}{2}\langle \bar{u}\gamma_5 d - \bar{d}\gamma_5 u \rangle$
- One of the charged pion becomes massless

Pion fields become linear combinations of $\bar{u}\gamma_5 d$, $\bar{d}\gamma_5 u$

Phase diagram: analysis $\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \rho = \frac{1}{2}\langle \bar{u}\gamma_5 d - \bar{d}\gamma_5 u \rangle$ as a function of T, μ_I, μ_B

Ladder-qcd at finite isospin chemical potential
A. Barducci, R. Casalbuoni, G. Pettini, L. R. (Phys. Lett. 2003)

Ladder qcd

- A renormalizable model (valid at high T, μ)
- Based on a variational approach: calculation of effective action, coupling a source to the composite operator $\bar{\psi}(x)\psi(y)$
- The physical system is studied at the minimum of the effective action
- At the minimum the effective action reproduces the free energy density of the system

The model allows the determination of

- Properties of octet mesons
- Phase diagram of the theory
- Thermodynamics $\leftrightarrow$ connection with statistical hadronization model

- Effective action for composite operators (in QCD $\bar{\psi}(x)\psi(y)$)

$$\Gamma[S] = -\text{Tr}\ \ln\left(S^{-1}\right) + \text{Tr}\left(S\frac{\delta\Gamma_2}{\delta S}\right) - \Gamma_2[S]$$

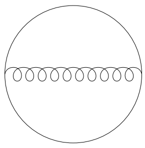
(Jackiw et al. 1974; Barducci et al. 1988)

- Γ_2 is the sum of all the vacuum two particle irreducible graphs (2PI) with exact propagator S

- Approximations: we perform a two loop calculation

$$\Gamma[S] = \ -\text{Tr}\ \ln(S^{-1}) + \Gamma_2$$

$$\Gamma_2 \ =$$



- We work in Landau gauge $S^{-1} = i\hat{p} - m_0 - \Sigma = S_0^{-1} - \Sigma$

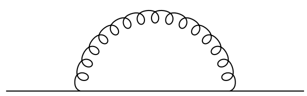
  no wave function renormalization

- Free gluon propagator

- With these two last hypothesis we can also take free vertex (Ward identity) $\rightarrow$ These hypothesis define ladder qcd

- Rigid coupling $g = g(p^2 = M^2)$ (we do not take into account corrections due to renormalization group)

- $\Sigma = -\dfrac{\delta\Gamma_2}{\delta S}$ is the dynamical variable, which is equal to the self-energy at the minimum of Γ



- $\Gamma[\Sigma] = \Gamma_{\log}[\Sigma] + \Gamma_2[\Sigma]$ $\Sigma = \Sigma_s + i\gamma_5\Sigma_p$ $\Sigma_s, \Sigma_p \in SU(3)$

- $\Gamma_2[\Sigma] = -2N_C\Omega_4 \frac{4\pi^2}{3g^2C_2} \int \frac{d^4p}{(2\pi)^4} \text{ tr } \left(\Sigma_s\Box\Sigma_s + \Sigma_p\Box\Sigma_p\right)$

- $\Gamma_{\log}[\Sigma] = -\text{Tr } \log[i\hat{p} - m - \Sigma_s(p^2) - i\gamma_5\Sigma_5(p^2)] =$
  $-\text{ Tr } \log(S_o^{-1} - \Sigma)$

- We have to find the minimum of the effective action with respect to the fields Σ. At the minimum the fields Σ is a constant

$$\Downarrow$$

$$\Gamma[\Sigma]|_{\text{Constant fields}} = V[\Sigma] \leftrightarrow \text{ Mean field approach}$$

Defining

- $\langle s_{ab}\rangle = -\frac{g^2}{3M^3}\langle\bar{\psi}_a\psi_b\rangle \quad ; \quad \langle p_{ab}\rangle = -\frac{g^2}{3M^3}\langle\bar{\psi}_a i\gamma_5\psi_b\rangle$

- The operator product expansion (OPE) tells us that
  $\Sigma(p^2) = (\mathbf{s} + i\gamma_5\mathbf{p})\ f(p^2)$
  $f(p^2) \sim 1/p^2$ for $p^2 \to \infty$

So, we have to introduce a suitable test function f that connects the self energy Σ with the fields s, p (matrices in flavor space)

$$\Downarrow$$

We have to make an Ansatz on its behaviour

Important:having introduced a test function f the functional derivatives become ordinary derivatives $\rightarrow$ we have to minimize the effective potential with respect to the constant fields s, p

The choose of the test function

In a variational approach, we want that the physical picture remains the same if we change the test function (the physics cannot depend on the Ansatz)

There are features that remain stable changing the Ansatz (phase diagram, properties of the mesons); however, to have a well-defined thermodynamics (equation of state)

$\Downarrow$

Good definition of the dispersion relation $E^2 \sim p^2 + \bar{\Sigma}^2$

$\Downarrow$

We define a class of test function for which the equation of state is well defined

$$\frac{f_N(x^2)}{M} = \frac{1 + x^2 + x^4 + ... + x^{2N-2}}{1 + x^2 + x^4 + ... + x^{2N-2} + x^{2N}} \quad x^2 = p^2/M^2$$

$N \geq 2 \Rightarrow$ free particle in the IR regime ($\Sigma \rightarrow$ constant)

The numerical computations gets harder when N is high: in this work we have used $N = 2$

Fit of the parameters $(c = \frac{2\pi^2}{g^2}, \widehat{m} = \frac{m_u + m_d}{2}, m_s, M)$ with the properties of the octet mesons $(T, \mu = 0)$

Extension at $T, \mu \neq 0$

- $\mu \neq 0$ $p^\nu \to (p^0 + i\mu, \vec{p})$ in $\hat{p}$ Dirac operator
- $\mu_I \neq 0$pion condensate$\rho = -\frac{g^2}{6M^3}(\langle \bar{u}\gamma_5 d - \bar{d}\gamma_5 u\rangle)$
  $\to$ u and d coupled
- We admit no formation of kaon condensate$\to$ strange is decoupled$\to$ study in SU(2) sector (u,d masses taken equal)
- $p^0 \to \omega_n = (2n+1)\pi T$ $\int dp^0 \to \sum_{n=-\infty}^{+\infty}$ Matsubara
- Test function depends on $(p^0)^2 + (\vec{p})^2$ $\to$
  at $T \neq 0$ depends on frequencies

-Tr log(S^{-1})=-$\int d^4p$ log det(S^{-1})=-$\sum_n \int d^3\vec{p}$ log det(S^{-1})

$$S^{-1} = \begin{pmatrix} i(\omega_n + i\mu_u)\gamma_0 + \vec{p}\cdot\vec{\gamma} - F_u & \rho f_2 \gamma_5 \\ -\rho f_2\gamma_5 & i(\omega_n + i\mu_d)\gamma_0 + \vec{p}\cdot\vec{\gamma} - F_d \end{pmatrix}$$

$F_a = m_a + f_2\chi_a$ $\chi_a = -\frac{g^2}{3M^3}\langle\bar{\Psi}_a\Psi_a\rangle$ $f_2 = f_{N=2}(p^2 = \omega_n^2 + \vec{p}^2)$

log det(S^{-1})=$\sum_{k=1}^{P}$log $(i\omega_n + z_k)$ for N=2 $\Rightarrow$ P=20

$\sum_{k=1}^{20}\int d^3\vec{p}$ $(\sum_{n=-\infty}^{+\infty}$log $(i\omega_n + z_k))$ $z_k = z_k(\vec{p}^2, \mu_B, \mu_I, \chi_u, \chi_d, \rho)$

$\Downarrow$

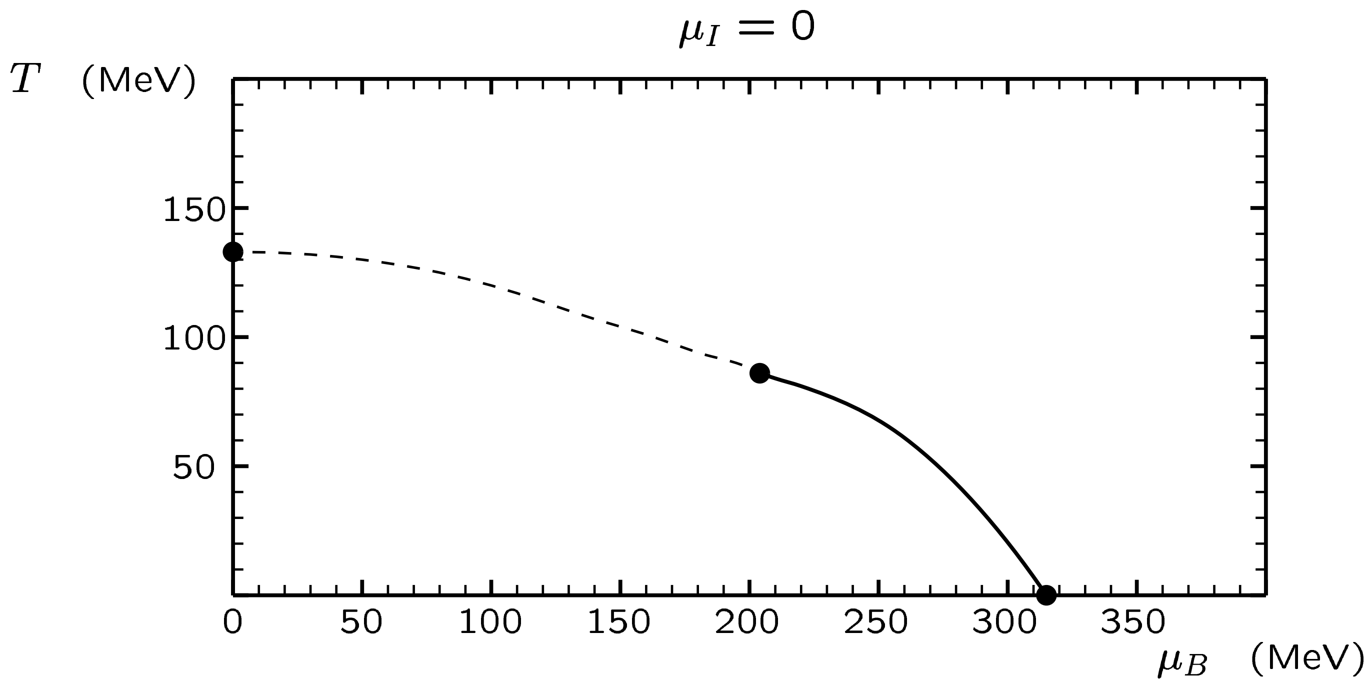
Effective potential V

Phase diagram

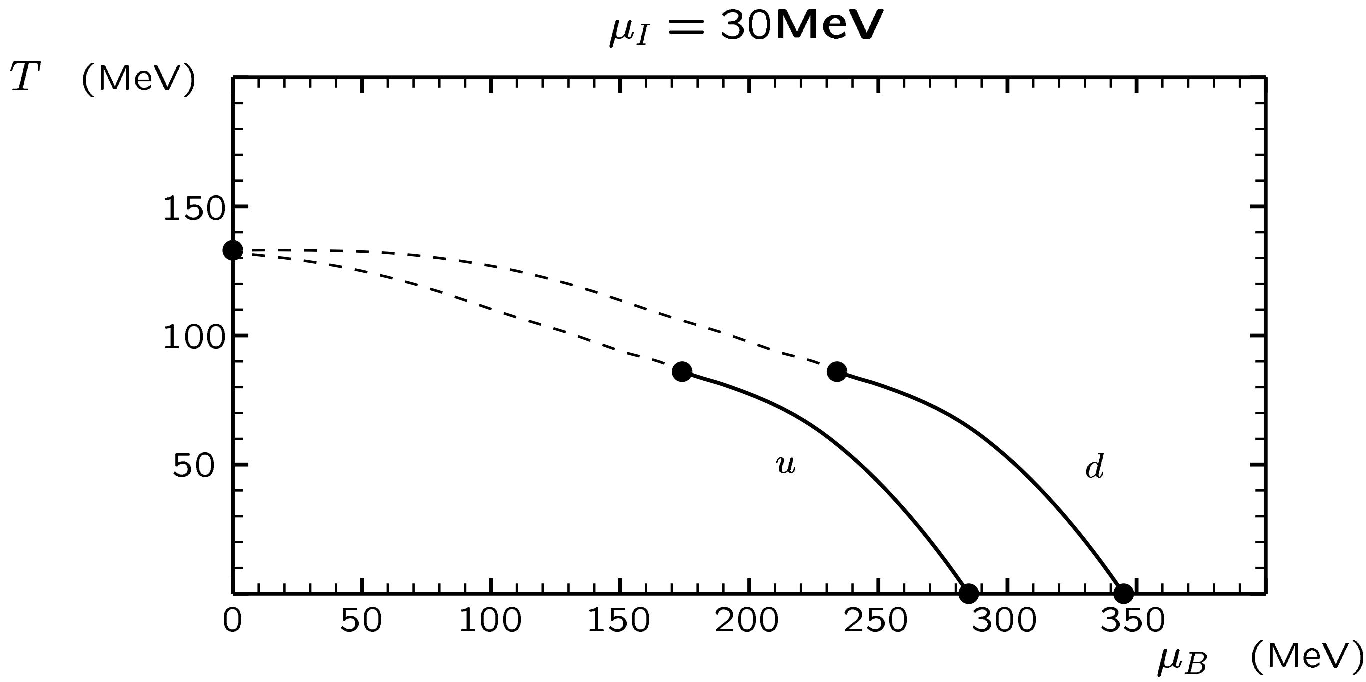
- $T, \mu_B, \mu_I \to \chi_u, \chi_d, \rho$ minima of effective potential
- Onerous task for the numerical computation
- The analysis of the whole space of parameters T, μ_B, μ_I has not yet been performed (numerical instabilities)

Results

- $\mu_I = 0$
  We have the "usual" phase diagram of the theory in the T, μ_B plane(first order phase transitions at low T, second order at high T, tricritical point)
- $T, \mu_B = 0$ $\mu_I > \mu_I^C = 70 MeV = m_\pi/2 \to \rho \neq 0$
- $T, \mu_B \neq 0$ $\mu_I^C(T,\mu) \geq \mu_I^C(T, \mu = 0)$
- There are critical values of T, μ_B over which the pion condensate never forms
- $T, \mu_B \neq 0$ $0 < \mu_I \leq 70$ MeV$\to$ u,d decoupled$\to$ two distinct phase transition
- The presence of an isospin chemical potential could make smoother the phase transition from qgp to hadronic phase
- We do not expect that the isospin chemical potential in H.I.C. can allow the formation of pion condensate



$\mu_I = 0$
T (MeV)
150
100
50
0
50
100
150
200
250
300
350
μ_B (MeV)



$\mu_I = 30$**MeV**
T (MeV)
150
100
50
0
50
100
150
200
250
300
350
μ_B (MeV)
u
d

Conclusions

- Study of an effective model of qcd (renormalizable)
- Good description of dynamical chiral symmetry breaking (as an effect of interactions)
- Good prediction for the phase diagram when $\mu_I = 0$
- Effect of the isospin chemical potential on the formation of a pion condensate and on the chiral transition
- Current work: study of the whole phase diagram
- Future task: inclusion of the di-quark fields