Thermodynamics of Large N_f QCD at Finite μ and Non-Fermi-Liquid Behavior

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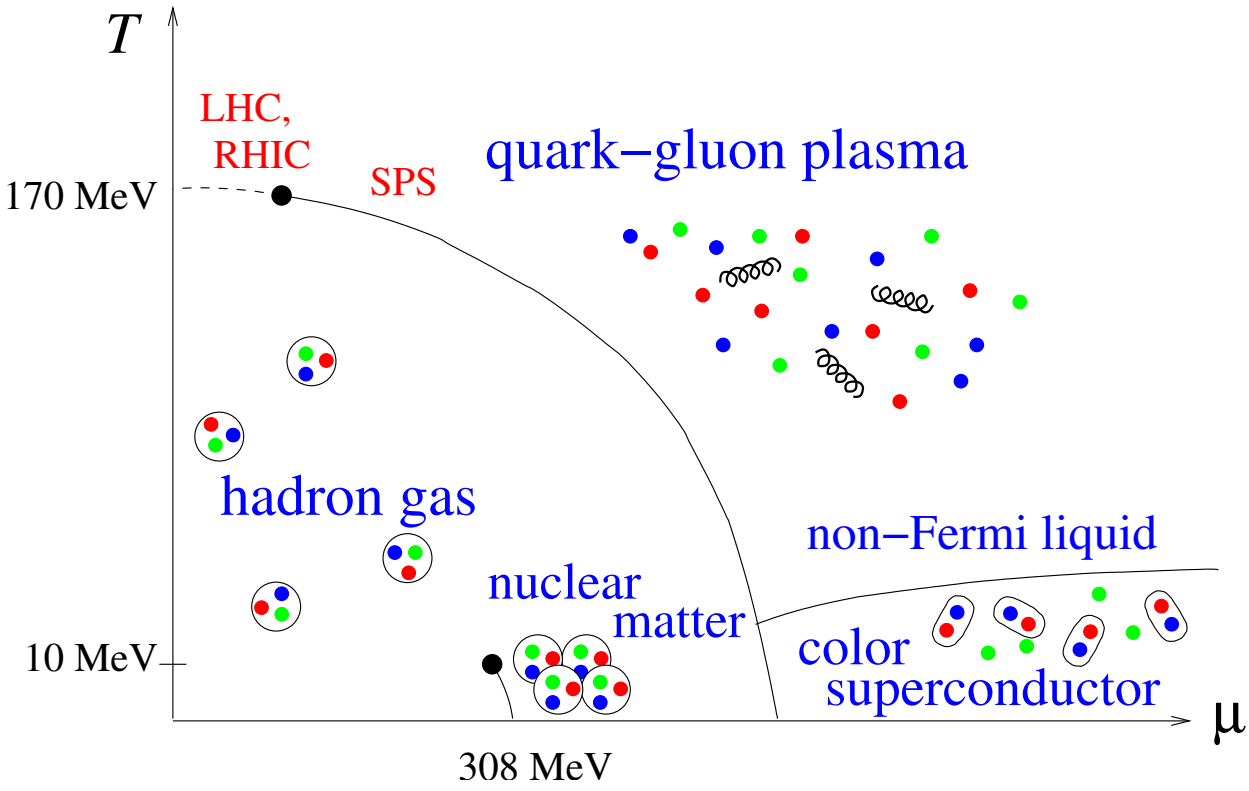
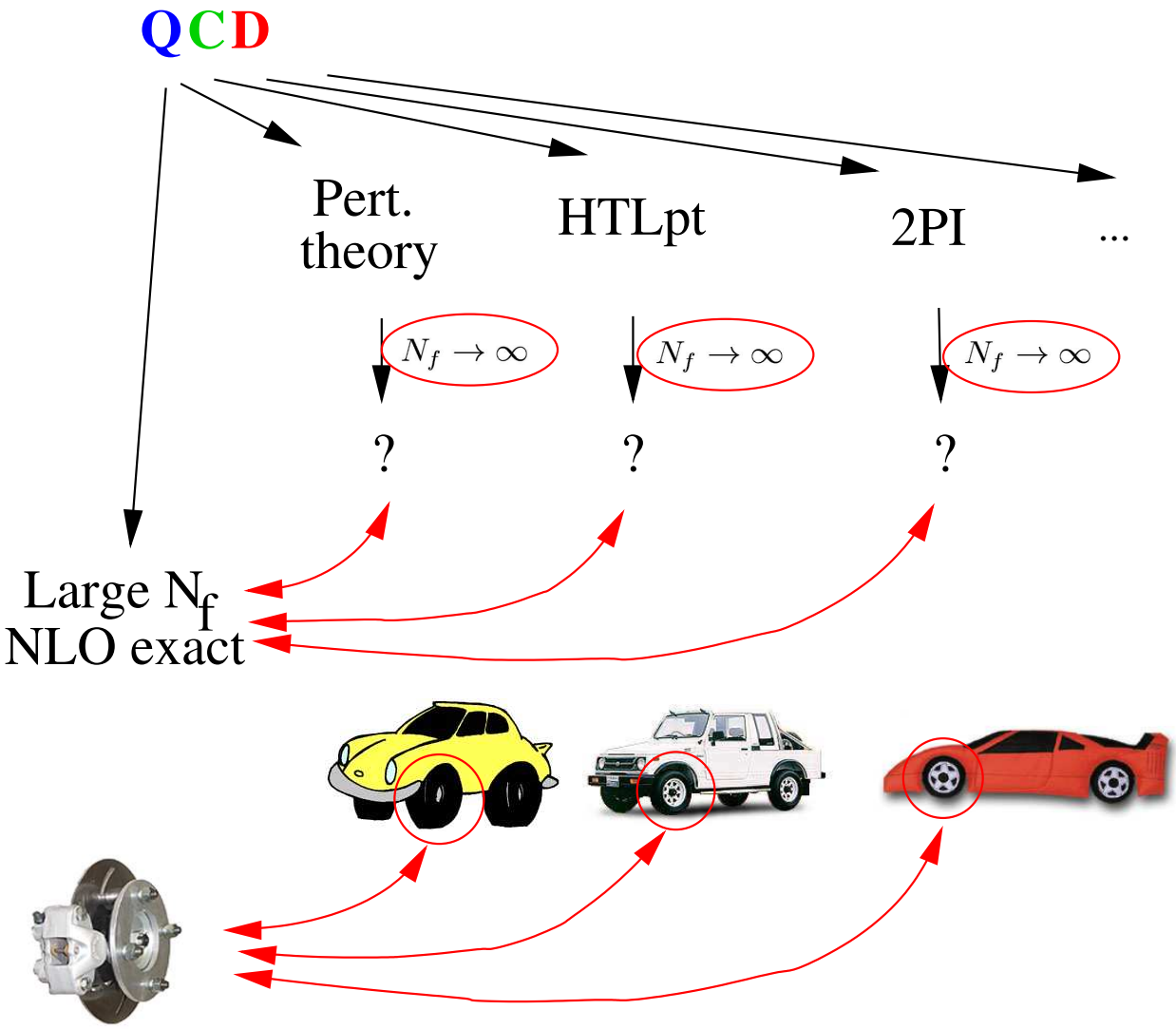
- Large N_f QCD: a test theory
- Finite μ
- High-density QCD:
  Non-Fermi liquid

Collaborateurs: Anton Rebhan, Andreas Gerhold

QCD Phase Diagram



Large N_f as a test theory



Pert. theory: Kajantie et al. 2002 + Ref therein
HTLpt: Andersen, Braaten, Strickland 2001
2PI: Blaizot, Iancu, Rebhan 1999

Perturbative calculations of QCD at high T/μ

$$P = \frac{8\pi^2}{45}T^4\left\{1 + \frac{21}{32}N_f\right.$$ Planck 1900

$$-\frac{15}{4}\left(1 + \frac{5}{12}N_f\right)\frac{\alpha_s}{\pi}$$ Shuryak/Chin 1978

$$+30\left(1 + \frac{N_f}{6}\right)^{\frac{3}{2}}\left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}}$$ Kapusta 1979

$$+\frac{135}{2}\left(1+\frac{N_f}{6}\right)\ln\frac{\alpha_s}{\pi}\left(1+\frac{N_f}{6}\right)\cdot\left(\frac{\alpha_s}{\pi}\right)^2$$ Toimela 1983

$$+\left\{237.2+15.97N_f-0.413N_f^2\right.$$ Arnold&Zhai 1995

$$\left.-\frac{165}{8}\left(1+\frac{5}{12}N_f\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar{\mu}}{2\pi T}\right\}\left(\frac{\alpha_s}{\pi}\right)^2$$

$$+\left(1+\frac{N_f}{6}\right)^{\frac{1}{2}}\left\{-799.2-21.96N_f-1.93N_f^2\right.$$ Zhai&Kastening 1995

$$\left.+\frac{495}{2}\left(1+\frac{N_f}{6}\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar{\mu}}{2\pi T}\right\}\left(\frac{\alpha_s}{\pi}\right)^{\frac{5}{2}}$$ Braaten&Nieto 1996

$$+\left\{1139.8+65.89N_f+7.653N_f^2\right.$$ Kajantie, Laine,

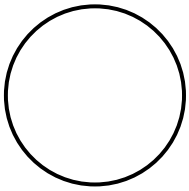
$$\left.-\frac{1485}{2}\left(1+\frac{N_f}{6}\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar{\mu}}{2\pi T}\right\}\ln\frac{\pi}{\alpha_s}\left(\frac{\alpha_s}{\pi}\right)^3$$ Rummukainen& Schröder 2002

$\mu_q \neq 0$: Vuorinen 2003

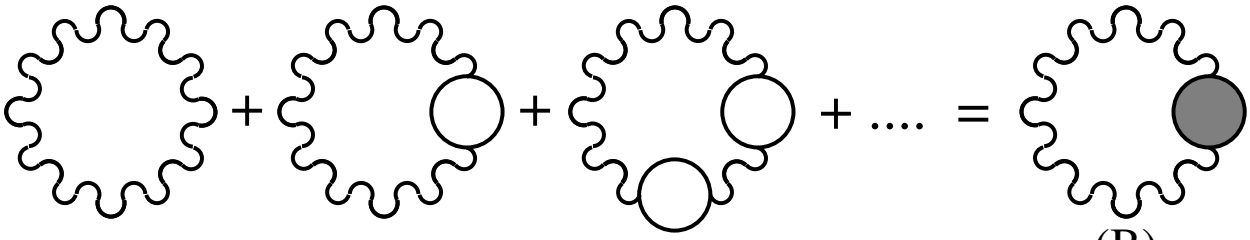
$$+\left\{?!+?N_f+?N_f^2+C(\bar{\mu})N_f^3\right\}\left(\frac{\alpha_s}{\pi}\right)^3+O\left(\frac{\alpha_s}{\pi}\right)^{\frac{7}{2}}\Bigg\}$$

* $C(\bar{\mu}) = 0.040...$for $\bar{\mu} = \pi T$ (A.I.&A.R. 2003)
* ?!: Completely non-perturbative
(Linde 1980; Gross, Pisarski & Yaffe 1980)

Pressure at Large N_f



(A)



(B)

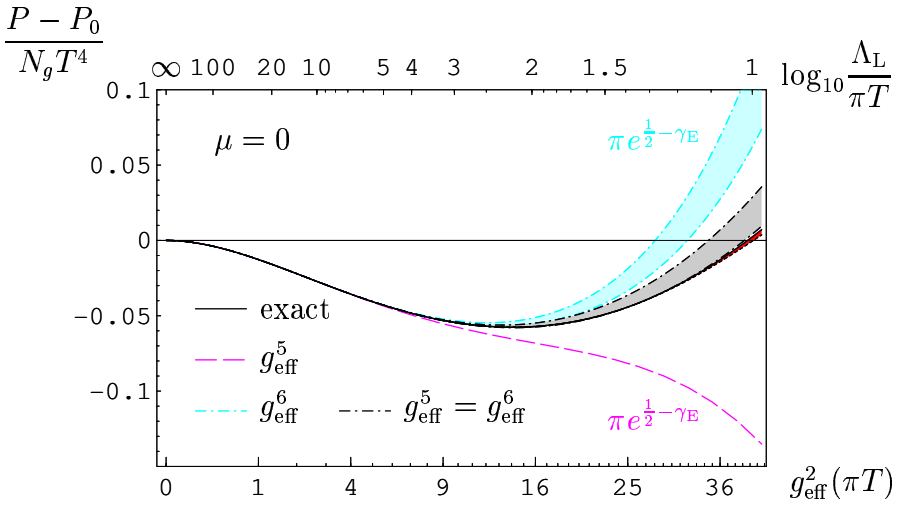
$$P_{\mathrm{LO}} = N N_f \left(\frac{\mu^4}{12\pi^2} + \frac{\mu^2 T^2}{6} + \frac{7\pi^2 T^4}{180} \right)$$

$$\begin{aligned} P_{\mathrm{NLO}} &= -N_g \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{dq_0}{\pi} \\ &\times \left[2 \left([n_b + \frac{1}{2}] \mathrm{Im}\,\mathrm{ln}\, D_T^{-1} - \frac{1}{2} \mathrm{Im}\,\mathrm{ln}\, D_{\mathrm{vac}}^{-1} \right) \right. \\ &+ \left. [n_b + \frac{1}{2}] \mathrm{Im}\,\mathrm{ln} \frac{D_L^{-1}}{q^2 - q_0^2} - \frac{1}{2} \mathrm{Im}\,\mathrm{ln} \frac{D_{\mathrm{vac}}^{-1}}{q^2 - q_0^2} \right] \end{aligned}$$

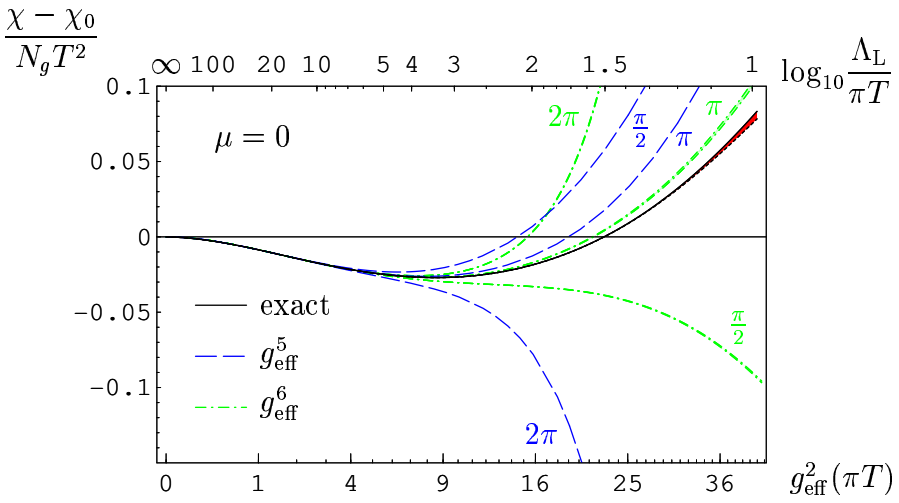
G.D.Moore, JHEP 0210 (hep-ph/0209190);
A.I., G.D.Moore, A.Rebhan, JHEP 0301 (hep-ph/0301057)

Large N_f NLO exactly solvable

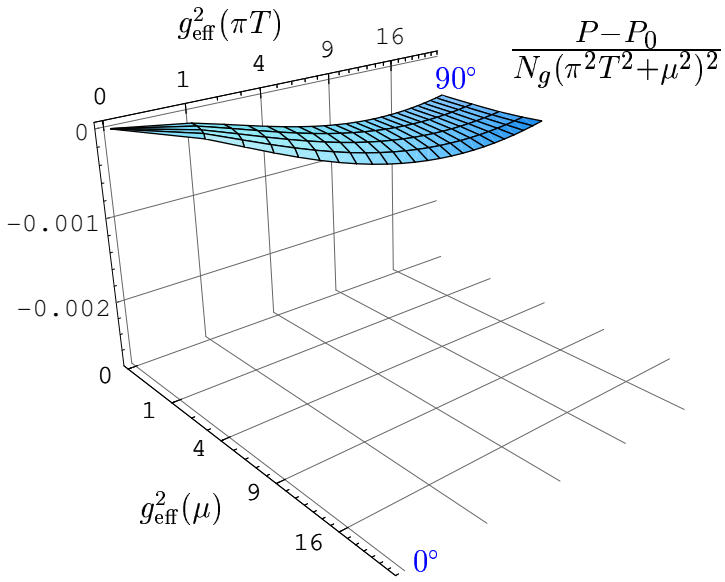
Pressure



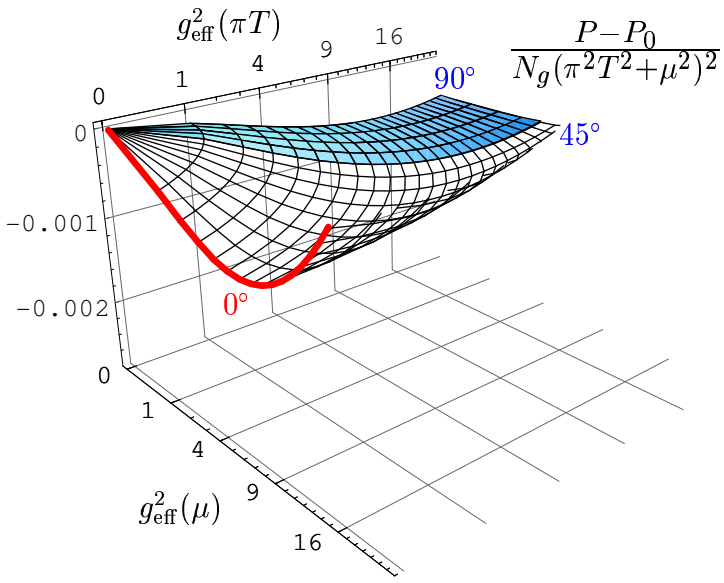
Quark Susceptibility $\chi = \frac{\partial N}{\partial \mu} = \frac{\partial^2 P}{\partial \mu^2}$



Large N_f at finite chemical potential

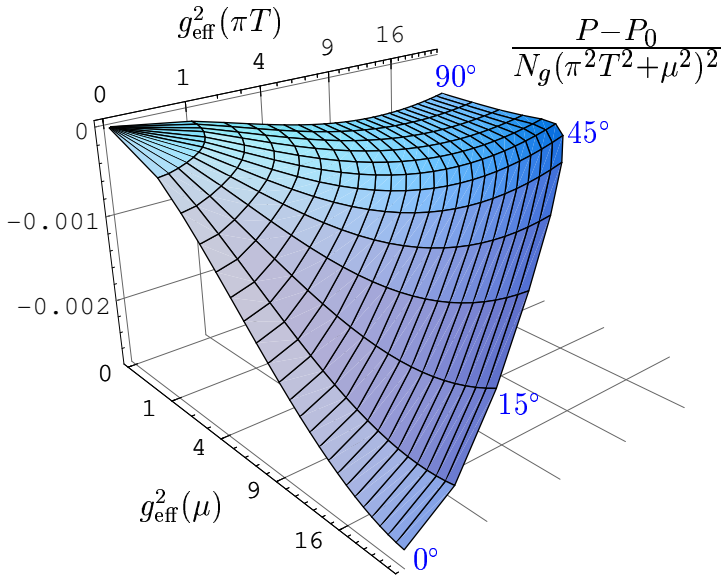


Large N_f at finite chemical potential



Naive continuation ???

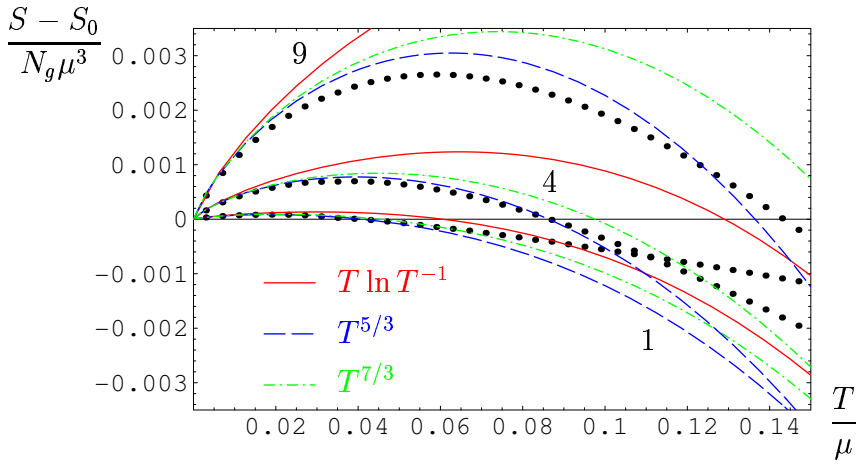
Large N_f at finite chemical potential



$\Rightarrow$ abrupt change for $\mu_q \gtrsim \pi T$

A.I.&A.Rebhan, JHEP (hep-ph/0305030)

Entropy for $T \ll \mu$: Non-Fermi-Liquid



$$\frac{S-S_0}{N_g} = \frac{g_{\rm eff}^2\mu^2 T}{36\pi^2}\left(\ln\frac{4g_{\rm eff}\mu}{\pi^2 T} - 2 + \gamma_E - \frac{6}{\pi^2}\zeta'(2)\right) + c_{5/3}T^{5/3} + c_{7/3}T^{7/3} + O(T^3)$$

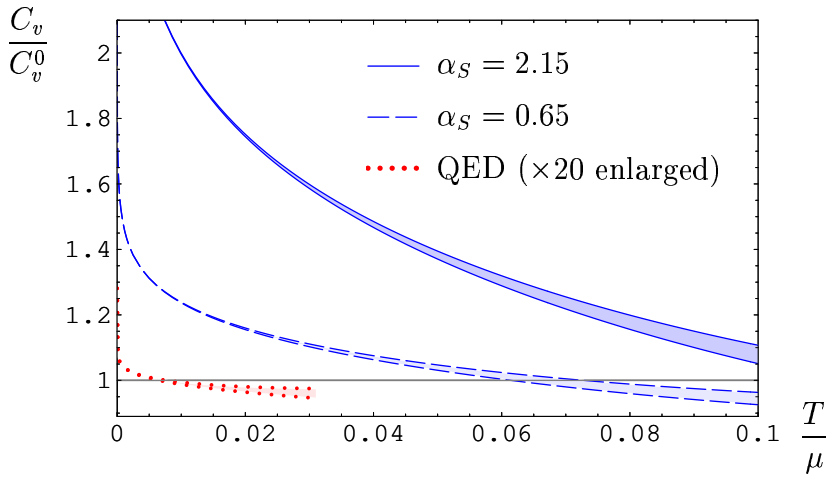
Fractional powers:

$$c_{5/3} = -\frac{8\,2^{2/3}(g_{\rm eff}\mu)^{4/3}\Gamma(\frac{8}{3})\zeta(\frac{8}{3})}{9\sqrt{3}\pi^{11/3}}$$

$$c_{7/3} = \frac{80\,2^{1/3}(g_{\rm eff}\mu)^{2/3}\Gamma(\frac{10}{3})\zeta(\frac{10}{3})}{27\sqrt{3}\pi^{13/3}}$$

A.I., A.Gerhold & A. Rebhan (hep-ph/0309019)

Specific heat for QED and QCD



$$\frac{C_V - C_V^0}{N_g} = \frac{g_{\rm eff}^2 \mu^2 T}{36\pi^2}\left(\ln\frac{4g_{\rm eff}\mu}{\pi^2 T} - 3 + \gamma_E - \frac{6}{\pi^2}\zeta'(2)\right) + \frac{5}{3}c_{5/3}T^{5/3} + \frac{7}{3}c_{7/3}T^{7/3} + O(T^3)$$

* normal quark matter component of neutron or proto-neutron stars

A.I., A.Gerhold & A. Rebhan (hep-ph/0309019)

Summary

- Large N_f QCD at NLO: a test theory that is exactly solvable

- Finite μ: Abrupt change for $\mu \gtrsim \pi T$ (hep-ph/0305030)

- High-density QCD:
  Non-Fermi liquid with fractional powers
  (hep-ph/0309019)