Numerical Simulations of Anisotropic Plasmas

Michael Strickland

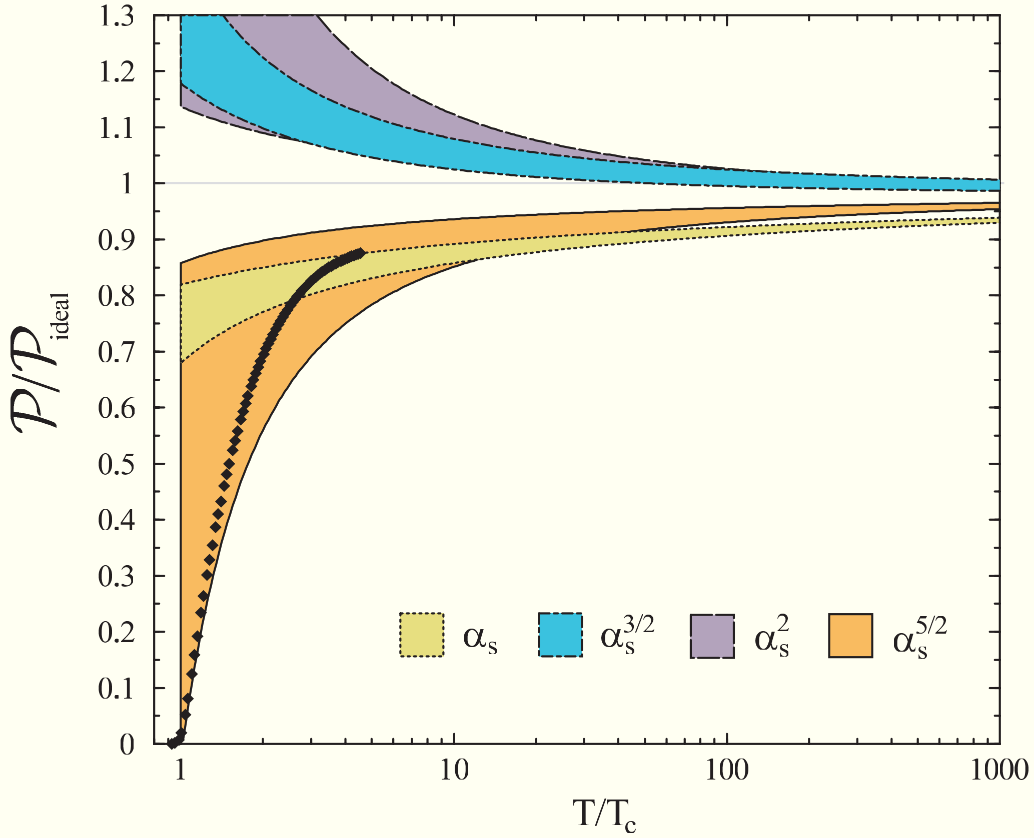
Frankfurt Institute for Advanced Studies

Paris-Bielefeld Block Course

16 March 2007

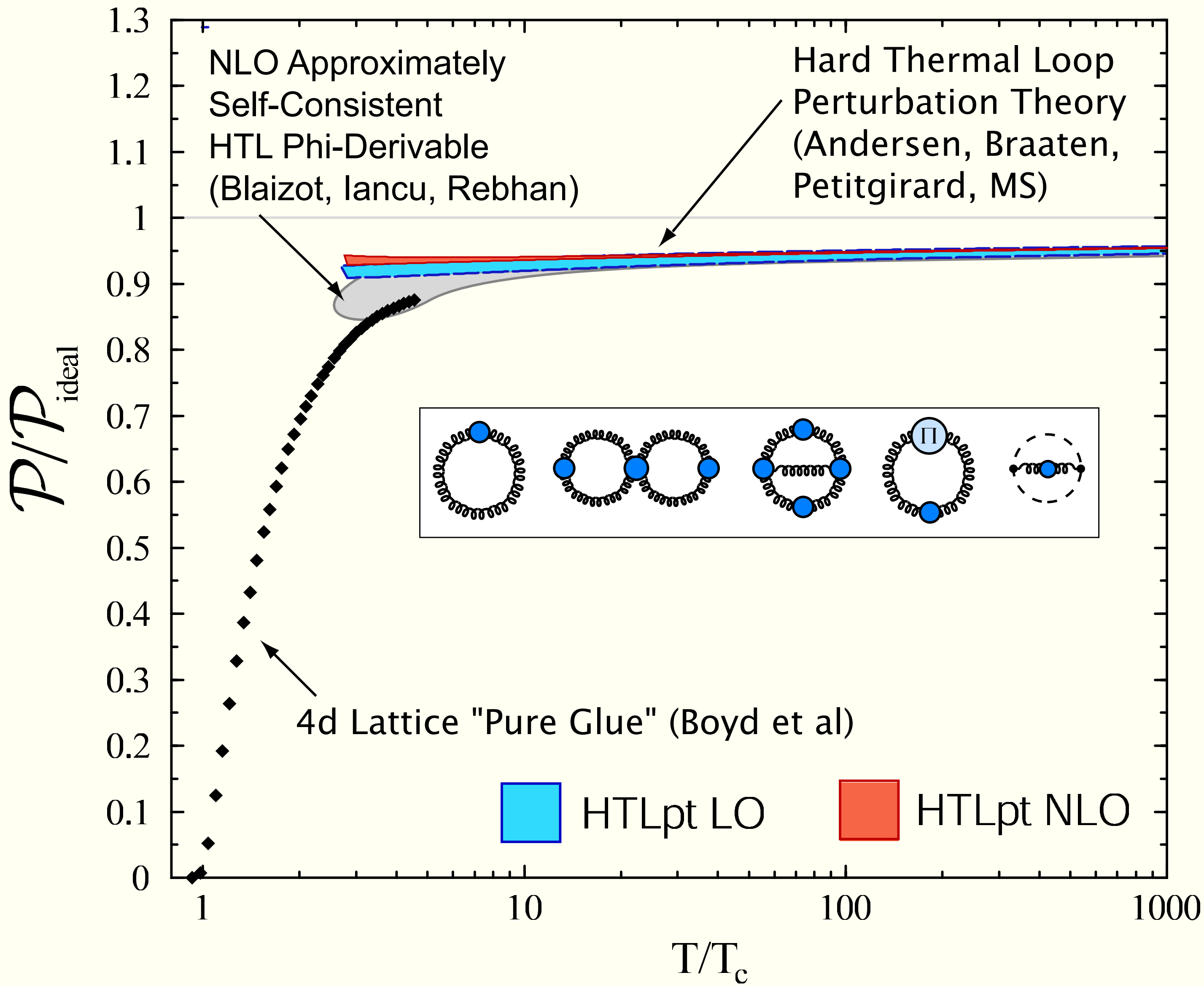
Cause for despair

Naive application of resummed finite-temperature perturbation theory to thermodynamics fails to converge at any reasonable temperature so should we abandon it?

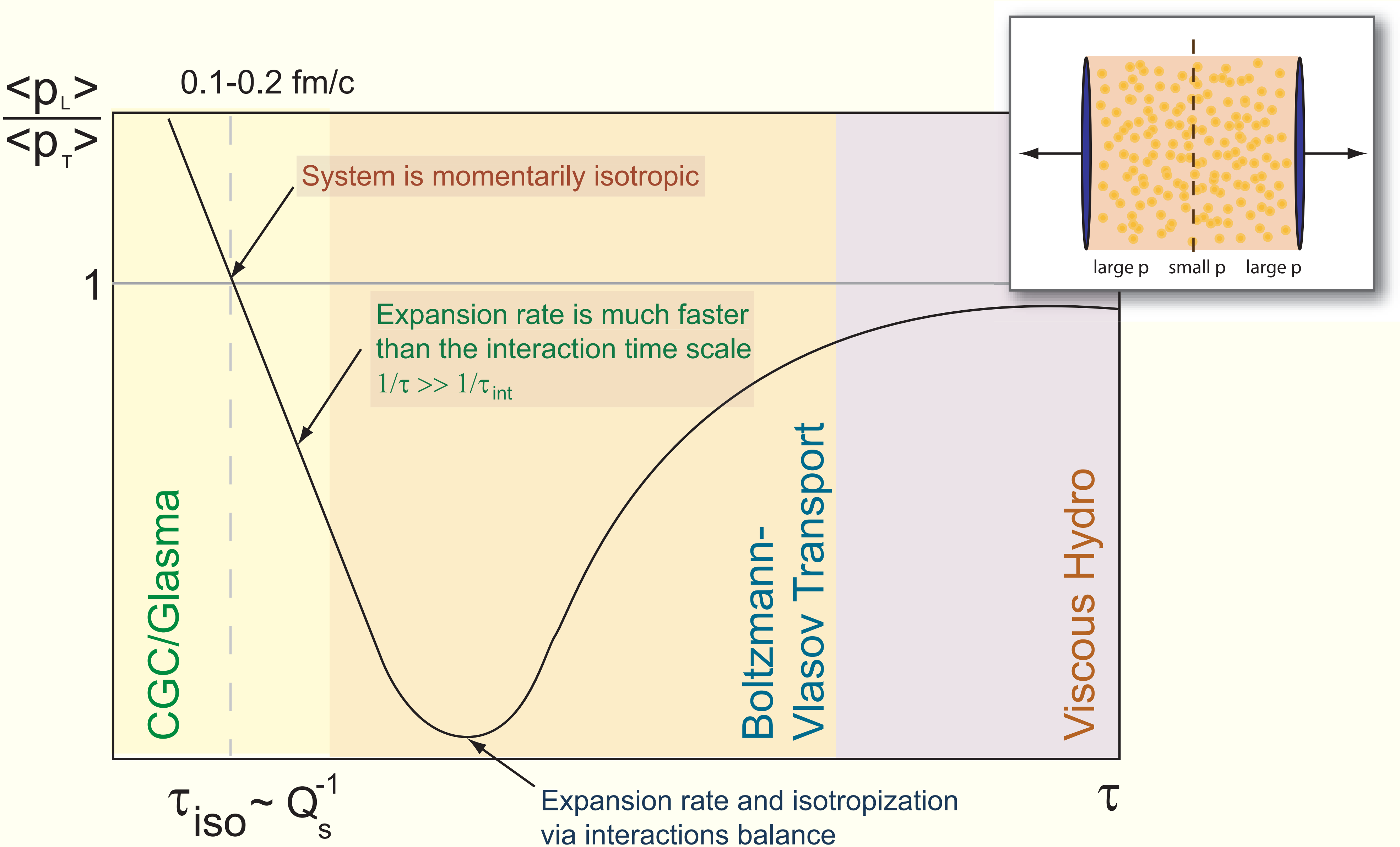


$$\mathcal{P}_{\rm QCD}/\mathcal{P}_{\rm ideal} = 1 - \frac{15}{4}\frac{\alpha_s}{\pi} + 30\left(\frac{\alpha_s}{\pi}\right)^{3/2}$$
$$+ \frac{135}{2}\left(\log\frac{\alpha_s}{\pi} - \frac{11}{36}\log\frac{\mu}{2\pi T} + 3.51\right)\left(\frac{\alpha_s}{\pi}\right)^2$$
$$+ \frac{495}{2}\left(\log\frac{\mu}{2\pi T} - 3.23\right)\left(\frac{\alpha_s}{\pi}\right)^{5/2}$$
$$+ \mathcal{O}(\alpha_s^3\log\alpha_s)$$

Cause for (limited) hope



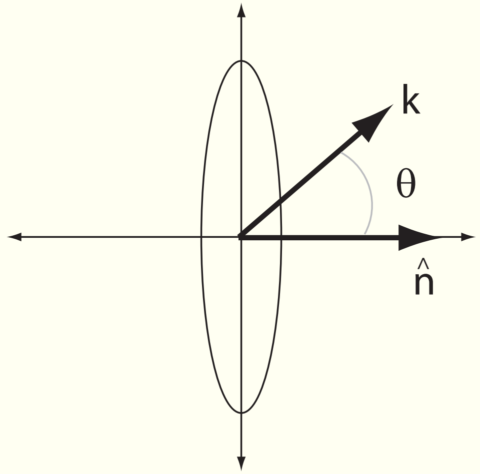
Momentum Space Anisotropy Time Dependence



The nature of the anisotropy

We assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument.

$$f(p^2) \rightarrow f(p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2)$$



The polarization tensor can then be written as

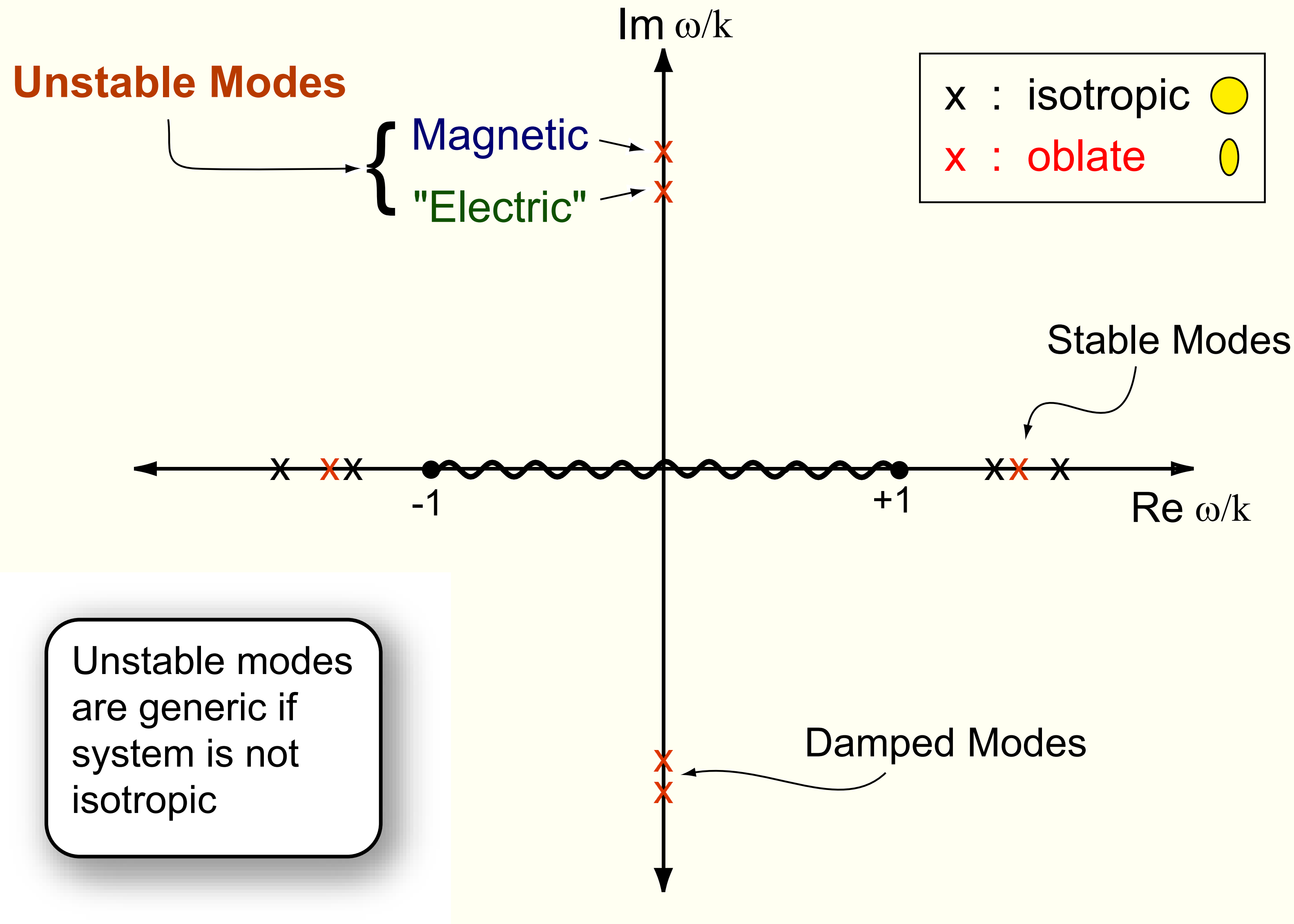
$$\Pi^{ij}_{ab}(\omega,k) = m_D^2\,\delta_{ab}\int\frac{d\Omega}{4\pi}v^i\frac{v^l+\xi(\mathbf{v}\cdot\mathbf{n})n^l}{(1+\xi(\mathbf{v}\cdot\mathbf{n})^2)^2}\left(\delta^{jl}-\frac{v^jk^l}{\omega-\mathbf{v}\cdot\mathbf{k}+i\epsilon}\right)$$

where m_D is the *isotropic* Debye mass

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\,p^2\frac{df(p^2)}{dp} \sim g^2 p_{\rm hard}^2$$

P. Romatschke and MS, hep-ph/0304092.

Anisotropic Gluonic Collective Modes ($\xi > 0$)

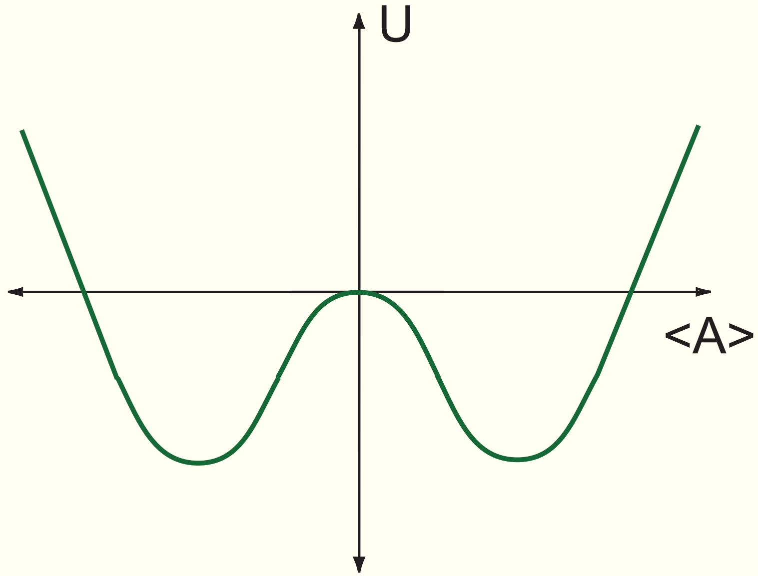


Anisotropic Hard-loop Effective Action

Require gauge invariance

$\downarrow$

Effective action for soft fields



$$S_{\text{soft}} = S_{\text{QCD}} + S_{\text{HL}}$$

$$S_{\text{HL}} = \frac{g^2}{2} \int_x \int_{\mathbf{p}} f(\mathbf{p})\, F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p \cdot D)^2} \right)_{ab} F_\rho{}^{b\,\mu}(x)$$
$$+ i \frac{C_F}{2} \tilde{f}(\mathbf{p}) \bar{\Psi}(x) \frac{p \cdot \gamma}{p \cdot D} \Psi(x) \Bigg\}$$

St. Mrówczyński, A. Rebhan, and MS, 2004

3+1 Real-Time Lattice Simulation (Pure Glue)

Numerically solve the equations of motion resulting from the hard-loop effective action on a space + velocity lattice.

$$j^\mu[A] = -g^2 \int_{\mathbf{p}} \frac{1}{2|\mathbf{p}|}\, p^\mu \,\frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})$$

with

$$[p \cdot D(A)] W_\beta(x;\mathbf{v}) = F_{\beta\gamma}(A) p^\gamma$$

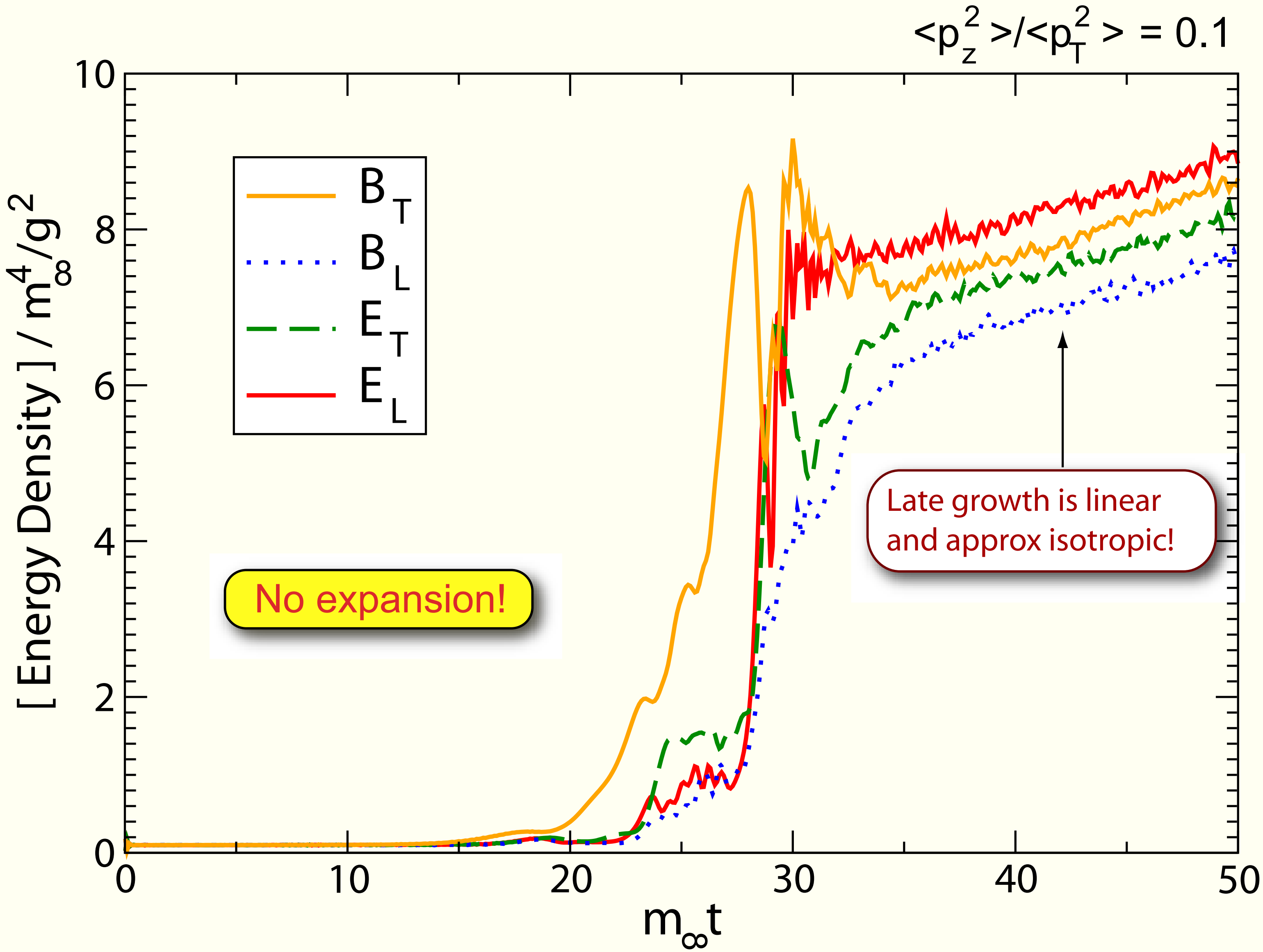
This has to be solved with the Yang-Mills equation

$$D_\mu(A) F^{\mu\nu} = j^\nu$$

where $\nu = 0$ is the Gauss law constraint.

Rebhan, Romatschke, and MS, 2005; Arnold, Moore, and Yaffe, 2005

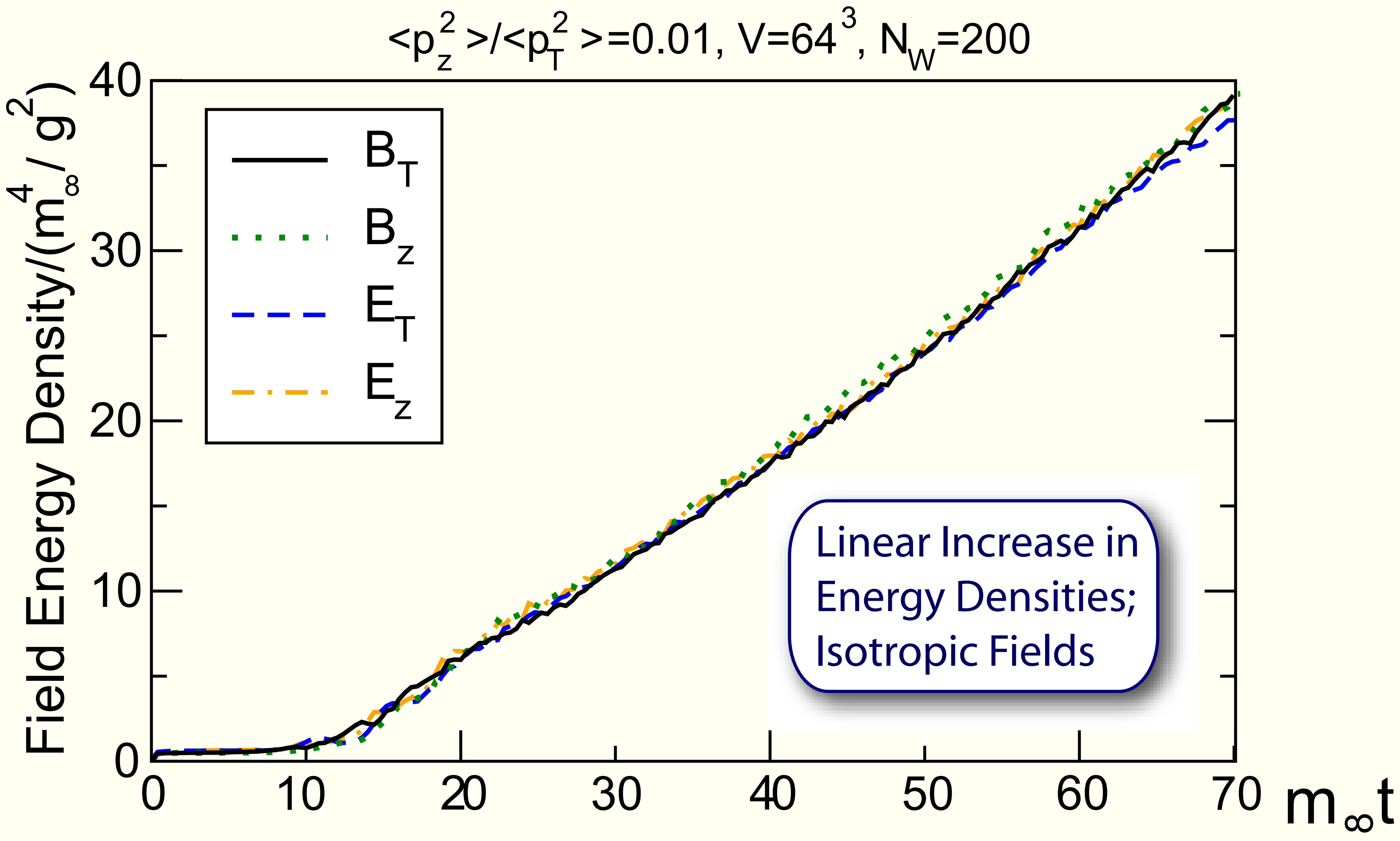
3D SU(2) Hard-Loop Results



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261

P. Arnold, G. Moore, and L. Yaffe, hep-ph/0505212

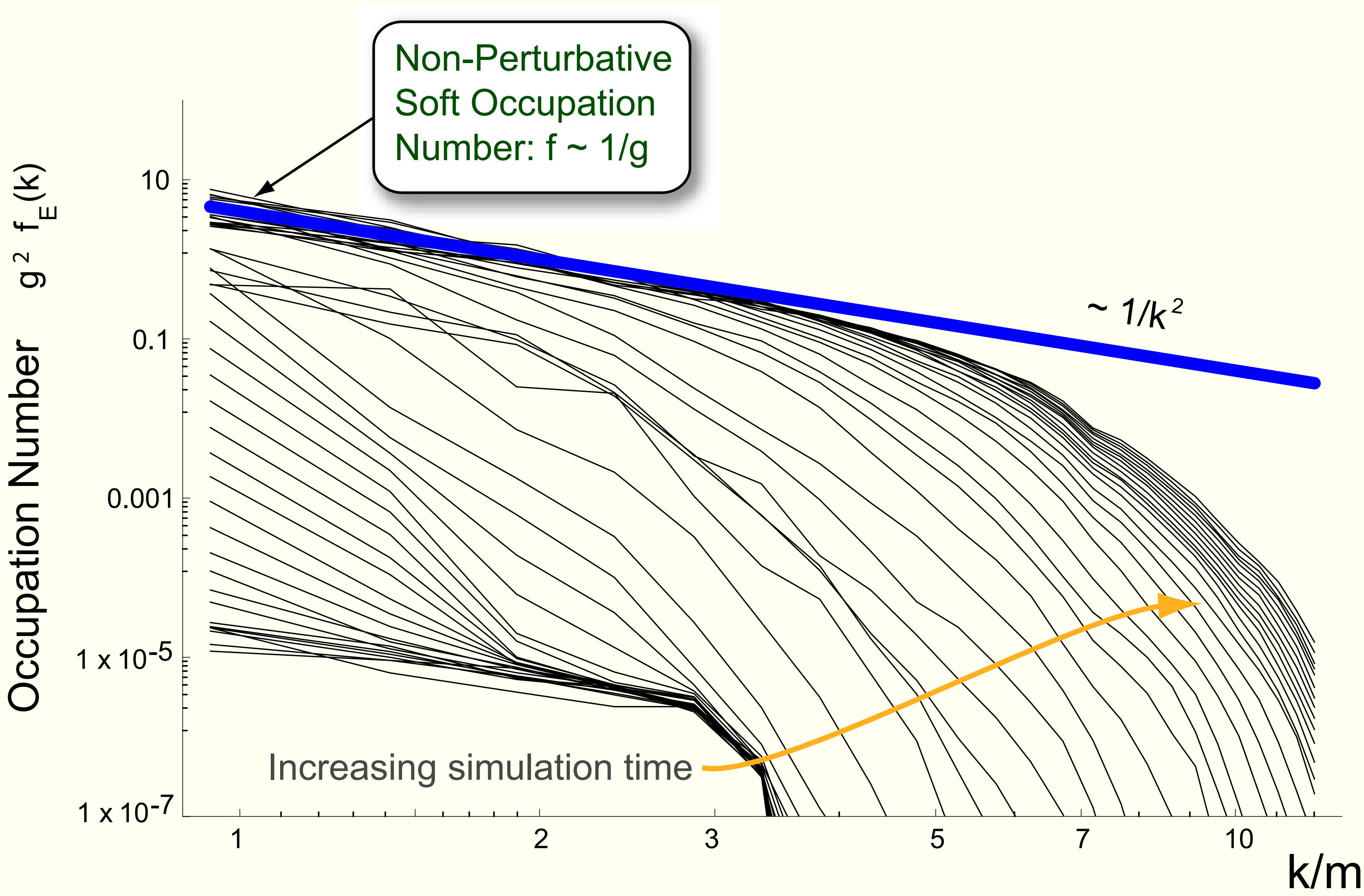
3D SU(2) Hard-Loop Results – Linear growth



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261

P. Arnold, G. Moore, and L. Yaffe, hep-ph/0505212

Kolmogorov cascade $\rightarrow$ Turbulent Fields



3D Colored-Particle-in-Cell Simulations (CPIC)

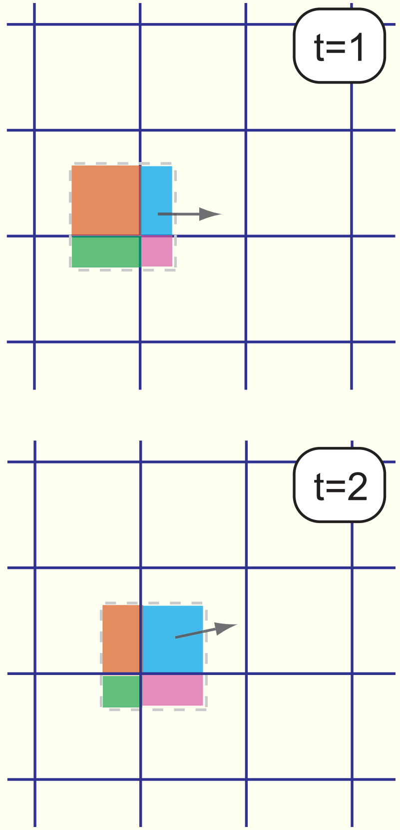
Hard-loop approximation strictly only applies when we ignore the back-reaction of the particles on their self-generated fields. How can we go beyond hard-loops?

Include back-reaction by solving collisionless transport equation without linearization

$$p^\mu[\partial_\mu - g q^a F^a_{\mu\nu}\partial^\nu_p - g f_{abc} A^b_\mu q^c \partial_{q^a}] f(x,p,q) = 0$$

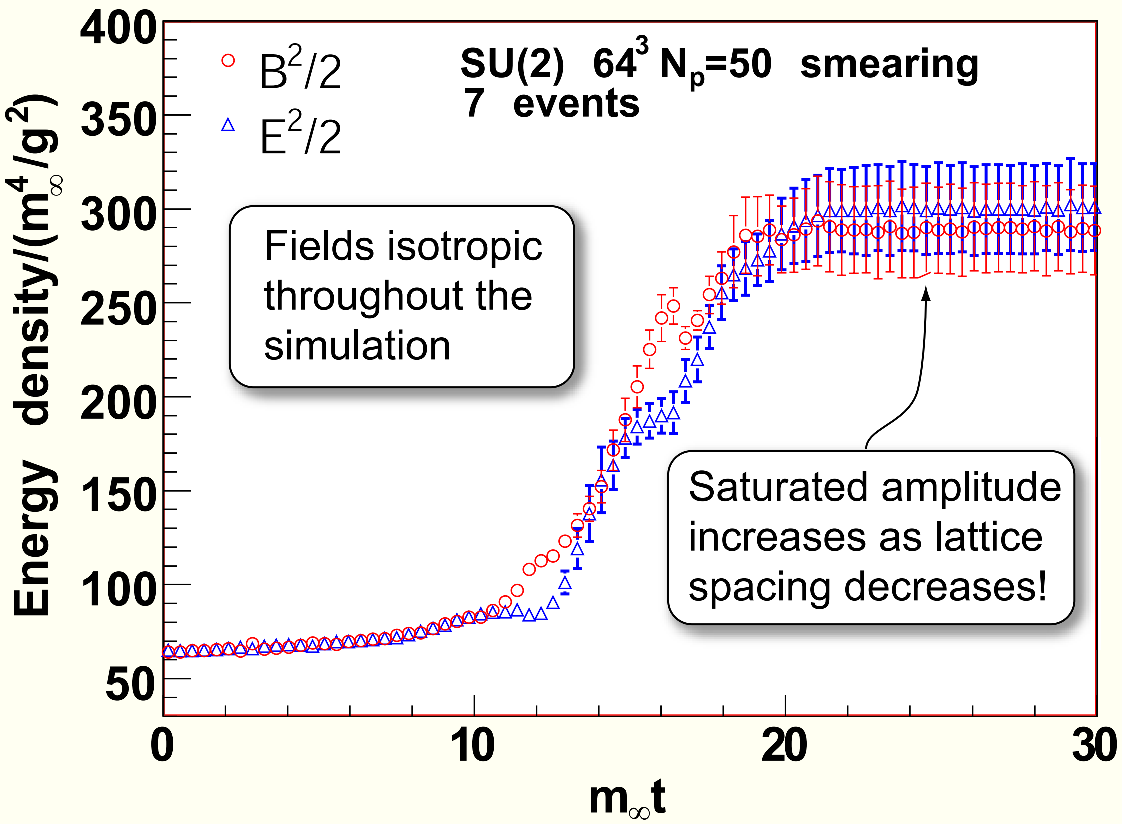
Coupled to the Yang-Mills equation for the soft gluon fields

$$D_\mu F^{\mu\nu} = J^\nu = g \int \frac{d^3p}{(2\pi)^3} dq\, q\, v^\nu f(t,\boldsymbol{x},\boldsymbol{p},q)$$

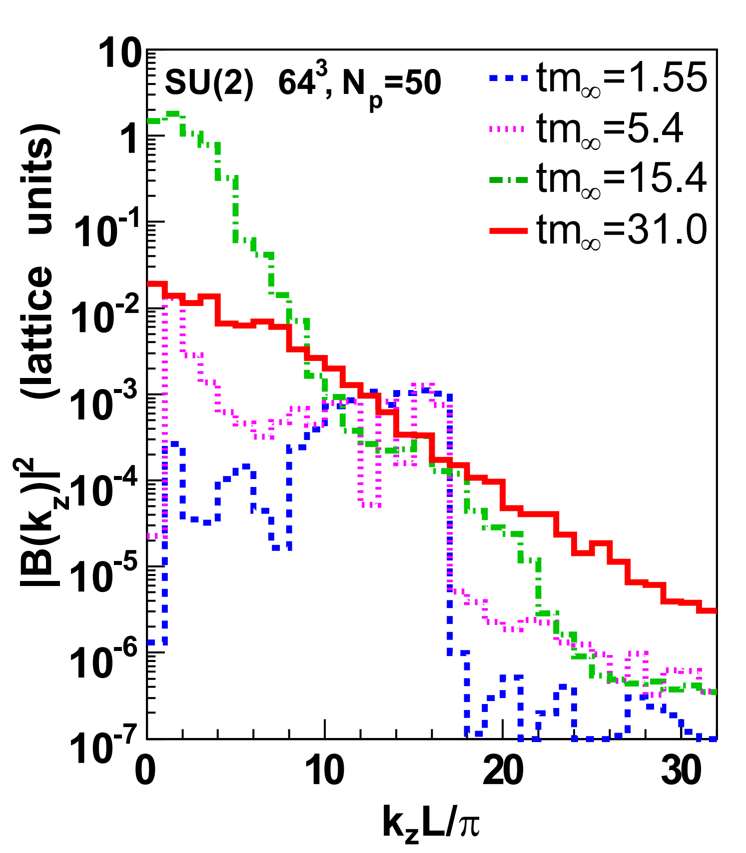


A. Dumitru, Y. Nara, and MS, hep-ph/0604149

CPIC Results – Ultraviolet Avalanche



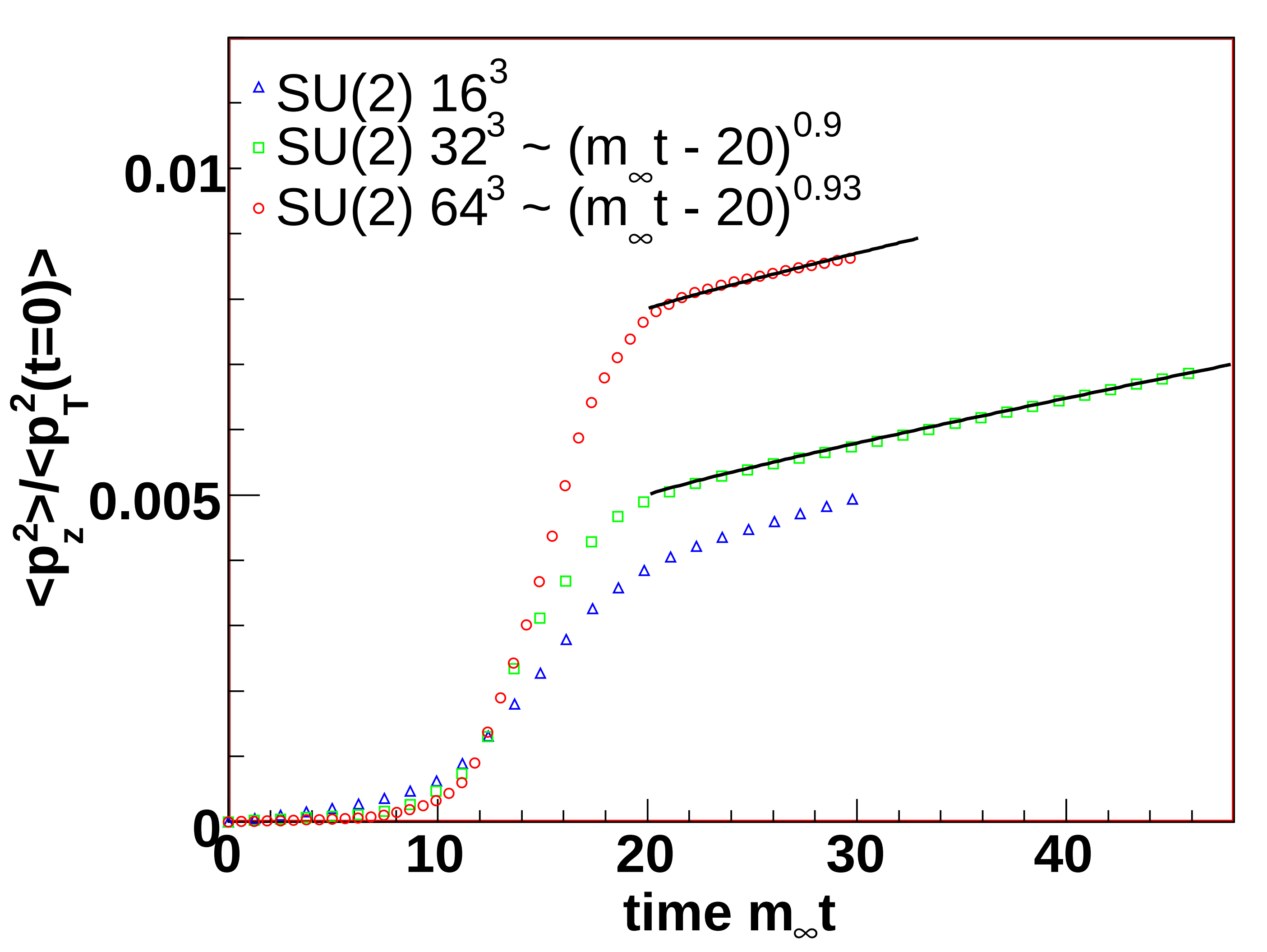
$L = 5$ fm, $p_{\rm hard} = 16$ GeV, $g^2\,n_g = 10/$fm^3, $m_\infty = 0.12$ GeV.



Coulomb gauge-fixed color-magnetic field spectrum at four different times.

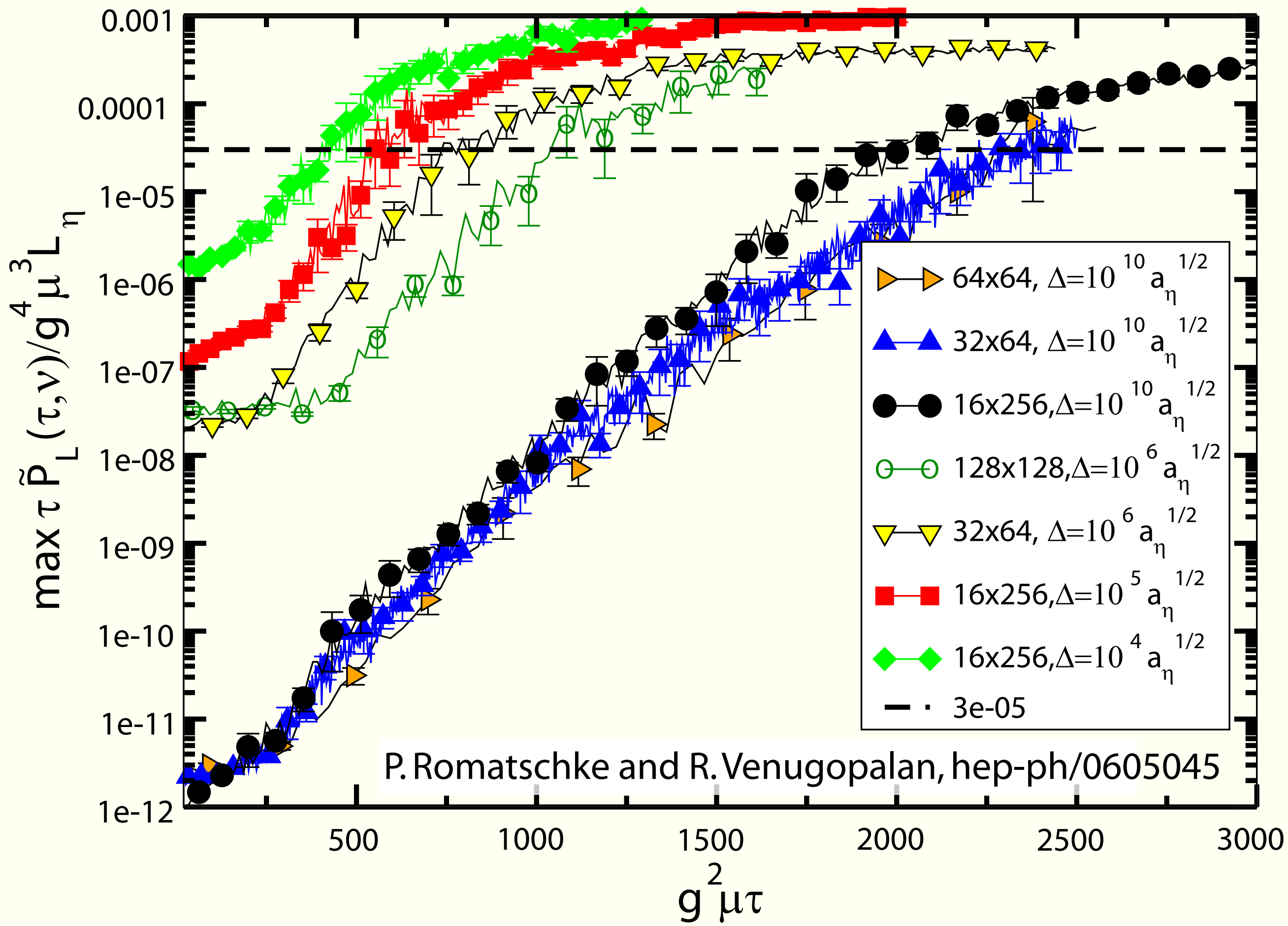
A. Dumitru, Y. Nara, and MS, hep-ph/0604149

CPIC Results – Particle Momentum



Instabilities in classical YM – The unstable glasma

Instabilities also seen in expanding classical Yang-Mills solutions which include rapidity fluctuations.



Growth $\sim e^{\sqrt{Q_s\tau}}$ agrees with HL calculation!

[P. Arnold, J. Lenaghan, and G. Moore, hep-ph/0307325]

[P. Romatschke and A. Rebhan, hep-ph/0605064]

Initial spectrum of rapidity fluctuations from CGC camp

[K. Fukushima, F. Gelis, and L. McLerran, hep-ph/0610416]

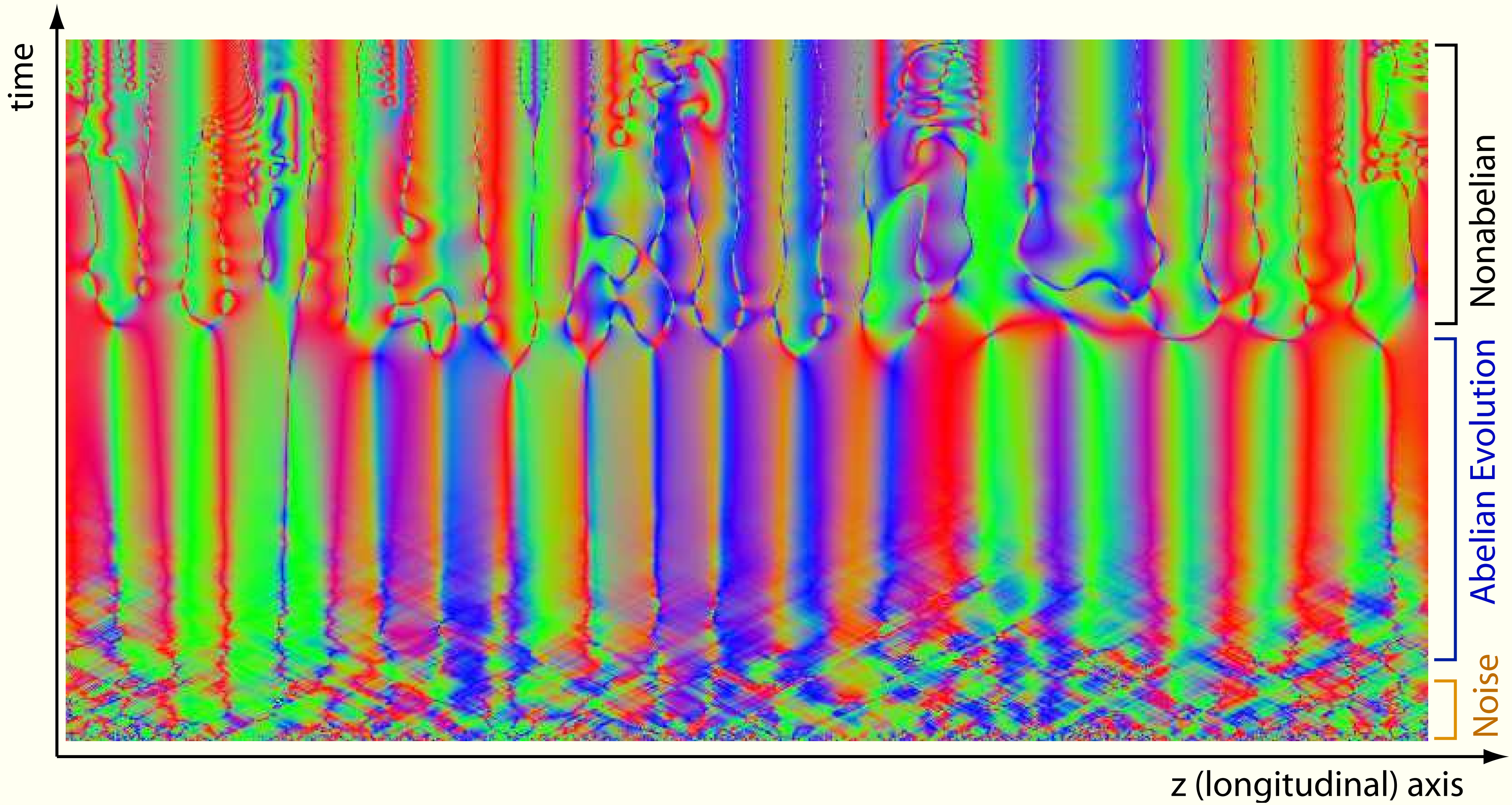
Conclusions

- Anisotropic plasmas are qualitatively different than isotropic ones.
- Hard-Loop : Fields show isotropic linear growth accompanied by cascade to UV.
- CPIC : Rapid isotropic field growth followed by UV “avalanche”.
- Classical YM : rapidity fluctuations $\rightarrow$ the “glasma” is unstable to becoming a QGP!
- Systematic calculations of p_T-p_L anisotropy observables such as jet effects and E&M signatures.

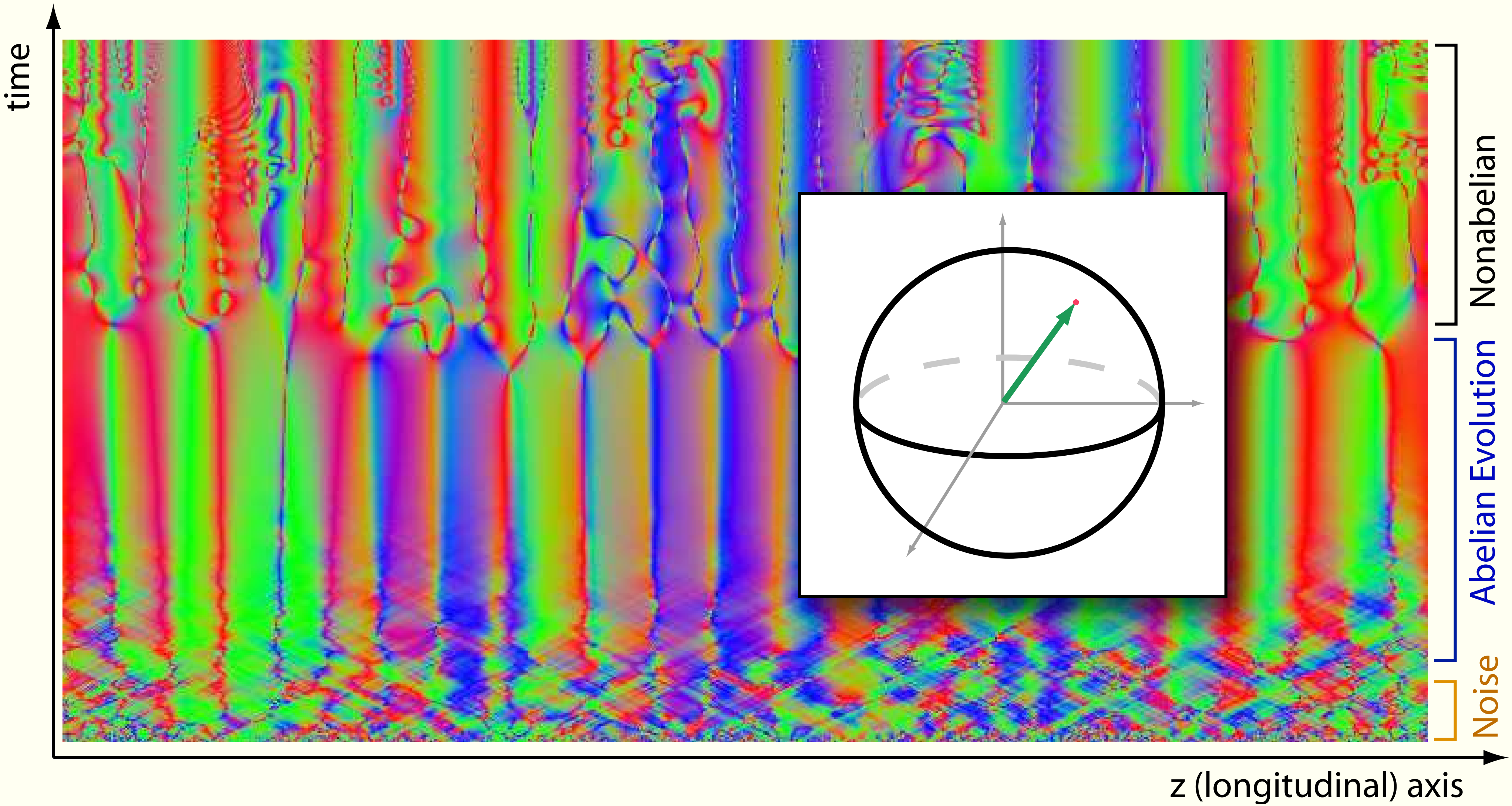
Hard-loop, classical YM, and CPIC simulations including realistic rapidity fluctuations in an expanding metric have yet to be performed.

. . . More hard work ahead . . .

SU(2) Visualization



SU(2) Visualization + Sphere



SU(2) Visualization + Zoom